Final Review

Solve the compound inequality and graph the solution set:

1) $-2x+5>7$ or $-3x+10>2x$

$-2x > 2$ $\quad\quad$ $-5x > -10$

$x < -2$ $\quad\quad$ $x < 2$

0 $\quad$ 2

Solve the inequality and graph the solution set:

2) $|x-4| \le 2$

$x-4=2$ $\quad\quad$ $x-4=-2$

$x=6$ $\quad\quad$ $x=2$

0 $\quad$ 2 $\quad$ 6

3) $6-\frac{2}{3}(3x-12)>\frac{2}{5}(10x+50)$

$6-2x+8>4x+20$

$-2x+14>4x+20$

$-6x>6$

$x<-1$

-1 $\quad$ 0

Simplify

4) $\sqrt[4]{x}\cdot\sqrt[5]{x}$

$x^{\frac{1}{4}}\cdot x^{\frac{1}{5}}$

$x^{\frac{5}{20}+\frac{4}{20}}$

$x^{\frac{9}{20}}$ $\quad$ $\sqrt[20]{x^9}$

5) $\left(36x^{\frac{1}{2}}y^{-\frac{1}{4}}\right)^{\frac{1}{2}}$

$6x^{\frac{1}{4}}y^{-\frac{1}{8}}$

$\frac{6x^{\frac{1}{4}}}{y^{\frac{1}{8}}}$

6) $\left(5\sqrt{6}+2\sqrt{2}\right)\left(\sqrt{6}+\sqrt{2}\right)$

$5(6)+5\sqrt{12}+2\sqrt{12}+2\cdot2$

$30+7\sqrt{12}+4$

$34+7\sqrt{4\cdot3}$

$34+14\sqrt{3}$

Solve the equation:

7) $7|5x|+2=16$

$7|5x|=14$

$|5x|=2$

$5x=2$ or $5x=-2$

$x=\frac{2}{5}$ or $x=-\frac{2}{5}$

8) $\frac{x+1}{3}+\frac{x+2}{7}=5$

$\frac{7(x+1)}{21}+\frac{3(x+2)}{21}=5$

$\frac{7x+7+3x+6}{21}=5$

$10x+13=21\cdot5$

$10x+13=105$

$10x=92$

$x=9.2$

Solve the equation

9) $\frac{x}{x+4} - 2 = \frac{11}{x^2-16}$ $(x-4)(x+4)$

$x(x-4) - 2(x^2-16) = 11(1)$

$x^2 + 4x - 2x^2 + 32 = 11$

$-x^2 - 4x + 32 = 11$

$x^2 + 4x - \cancel{43}\,21 = 0$

$(x+7)(x-3) = 0$

$x = -7, 3$

10) $\sqrt{x+9} - \sqrt{x-7} = 2$

$\sqrt{x+9} = 2 + \sqrt{x-7}$

$x + 9 = 4 + 4\sqrt{x+7} + x - 7$

$x + 9 = 4\sqrt{x-7} + x - 3$

$12 = 4\sqrt{x-7}$

$3 = \sqrt{x-7}$

$9 = x - 7$

$16 = x$

11) $\log_6(4x-1) = \frac{1}{2}$

$6^{\frac{1}{2}} = 4x - 1$

$1 + \sqrt{6} = 4x$

$x = \frac{1+\sqrt{6}}{4}$

12) Use the properties of logarithms to expand the logarithmic expression as much as possible. Where possible, evaluate the logarithmic expression: $\log_5\left(\sqrt[3]{\frac{x^2y}{25}}\right)$

$\frac{1}{3}\log_5 \frac{x^2y}{25}$

$\frac{1}{3}\left(\log_5 x^2y - \log_5 25\right)$

$\frac{1}{3}\left(2\log_5 x + \log_5 y - 5\right)$

$\frac{2}{3}\log_5 x + \frac{1}{3}\log_5 y - \frac{5}{3}$

13) Rationalize the denominator: $\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$ $\left(\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right)$

$\frac{2 + 2\sqrt{6} + 3}{2 - 3}$ $\frac{5 + 2\sqrt{6}}{-1}$

$\cancel{-5+}$ $-5 - 2\sqrt{6}$

14) If f(x) = $\frac{2x-5}{4}$, find $f^{-1}(x)$

$y = \frac{2x-5}{4}$

$x = \frac{2y-5}{4}$

$4x = 2y - 5$

$2y = 4x + 5$

$y = \frac{4x+5}{2}$

$f^{-1}(x) = \frac{4x+5}{2}$

15) If f(x) = $x^2 + x$ and g(x) = 3x + 1, find $(g \circ f)(x)$

$g(f(x))$

$g(x^2+x) = 3(x^2+x) + 1$

$3x^2 + 3x + 1$

$(g \circ f)(x) = 3x^2 + 3x + 1$

16) Find the distance between the points (6, 3) and (4, 1)

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$d = \sqrt{(4-6)^2 + (1-3)^2}$

$d = \sqrt{(-2)^2 + (-2)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$

$d = 2\sqrt{2}$

17) Write the equation of a circle in standard form if the endpoints of the diameter of the circle are (4, 6) and (- 6, -2)

$d = \sqrt{(-6-4)^2 + (-2-6)^2}$

$= \sqrt{(-10)^2 + (-8)^2}$

$= \sqrt{100+64}$

$= \sqrt{164}$

$= \sqrt{4 \cdot 41}$ $2\sqrt{41}$

radius $= \sqrt{41}$

center $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{4+6}{2}, \frac{6+2}{2}\right)$

$= \left(\frac{10}{2}, \frac{8}{2}\right)$

$= (5, 4)$

$(x-5)^2 + (y-4)^2 = 41$

18) If the given sequence is arithmetic, find the sum of the first 30 terms. If the sequence is geometric, find the sum of the first 8 terms.

a) 3, 9, 27, 81, . . .

$\frac{9}{3} = 3$

$\frac{27}{3} = 3$

$r = 3$

geometric

$S_8 = \frac{a_1(1-r^n)}{1-r}$

$= \frac{3(1-3^8)}{1-3}$

$= \frac{3(1-6561)}{-2}$

$= \frac{3(-6560)}{-2}$

$= \frac{-19680}{-2}$

$S_8 = 9840$

b) 2, 7, 12, 17, . . .

$7-2 = 5$

$12-7 = 5$

$d = 5$

$a_{30} = a_1 + (n-1)d$

$= 2 + (30-1)5$

$= 2 + (29)5$

$= 2 + 145$

$a_{30} = 147$

$S_{30} = a_1 + (n-1)d$

$= 2 + (30-1)5$

$= 2 + (29)5$

$= 2 + 145$

$S_{30} = 147$