

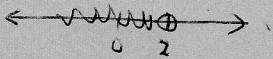
## Final Review

Solve the compound inequality and graph the solution set:

$$1) -2x + 5 > 7 \text{ or } -3x + 10 > 2x$$

$$-2x > 2 \quad -5x > -10$$

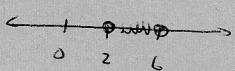
$$x < -1 \quad x < 2$$



Solve the inequality and graph the solution set:

$$2) |x - 4| \leq 2$$

$$x - 4 = -2 \quad x - 4 = 2 \\ x = 6 \quad x = 2$$



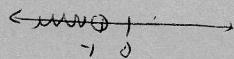
$$3) 6 - \frac{2}{3}(3x - 12) > \frac{2}{5}(10x + 50)$$

$$6 - 2x + 8 > 4x + 20$$

$$-2x + 14 > 4x + 20$$

$$-6x > 6$$

$$x < -1$$



Simplify

$$4) \sqrt[4]{x} \cdot \sqrt[5]{x}$$

$$\begin{aligned} &x^{\frac{1}{4}} \cdot x^{\frac{1}{5}} \\ &x^{\frac{5}{20}} \cdot x^{\frac{4}{20}} \\ &x^{\frac{9}{20}} \quad \sqrt[20]{x^9} \end{aligned}$$

$$5) \left( 36x^{\frac{1}{2}}y^{-\frac{1}{4}} \right)^{\frac{1}{2}}$$

$$\begin{aligned} &6x^{\frac{1}{4}}y^{-\frac{1}{8}} \\ &\frac{6x^{\frac{1}{4}}}{y^{\frac{1}{8}}} \end{aligned}$$

$$6) (5\sqrt{6} + 2\sqrt{2})(\sqrt{6} + \sqrt{2})$$

$$5(6) + 5\sqrt{12} + 2\sqrt{12} + 2 \cdot 2$$

$$30 + 7\sqrt{12} + 4$$

$$34 + 7\sqrt{48}$$

$$34 + 14\sqrt{3}$$

Solve the equation:

$$7) 7|5x| + 2 = 16$$

$$7|5x| = 14$$

$$|5x| = 2$$

$$5x = 2 \text{ or } 5x = -2$$

$$x = \frac{2}{5} \text{ or } x = -\frac{2}{5}$$

$$8) \frac{x+1}{3} + \frac{x+2}{7} = 5$$

$$\frac{7(x+1)}{21} + \frac{3(x+2)}{21} = 5$$

$$\frac{7x+7 + 3x+6}{21} = 5$$

$$10x + 13 = 210$$

$$10x + 13 = 210$$

$$10x = 197$$

$$x = 19.7$$

Solve the equation

$$9) \frac{x}{x+4} - 2 = \frac{11}{x^2 - 16} \quad (x-4)(x+4)$$

$$x(x-4) - 2(x^2 - 16) = 11(4)$$

$$x^2 + 4x - 2x^2 + 32 = 44$$

$$-x^2 - 4x + 32 = 44$$

$$x^2 + 4x - 43 = 0$$

$$11) \log_6(4x-1) = \frac{1}{2}$$

$$6^{\frac{1}{2}} = 4x - 1$$

$$1 + \sqrt{6} = 4x$$

$$x = \frac{1 + \sqrt{6}}{4}$$

12) Use the properties of logarithms to expand the logarithmic expression as much as possible. Where possible, evaluate the logarithmic expression:  $\log_5\left(\sqrt[3]{\frac{x^2y}{25}}\right)$

$$\frac{1}{3} \log_5 \frac{x^2y}{25}$$

$$\frac{1}{3} (\log_5 x^2 y - \log_5 25)$$

$$\frac{1}{3} (2 \log_5 x + \log_5 y - 5)$$

$$\frac{2}{3} \log_5 x + \frac{1}{3} \log_5 y - \frac{5}{3}$$

$$13) \text{ Rationalize the denominator: } \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} - \sqrt{3}} \quad \left( \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} \right)$$

$$\frac{2 + 2\sqrt{6} + 3}{2 - 3} \quad \frac{5 + 2\sqrt{6}}{-1}$$

~~$$= 5 + 2\sqrt{6}$$~~

$$14) \text{ If } f(x) = \frac{2x-5}{4}, \text{ find } f^{-1}(x)$$

$$y = \frac{2x-5}{4}$$

$$x = \frac{2y-5}{4}$$

$$4x = 2y - 5$$

$$2y = 4x + 5$$

$$y = \frac{4x+5}{2}$$

$$f^{-1}(x) = \frac{4x+5}{2}$$

$$15) \text{ If } f(x) = x^2 + x \text{ and } g(x) = 3x + 1, \text{ find } (g \circ f)(x)$$

$$g(f(x))$$

$$g(x^2 + x) = 3(x^2 + x) + 3x^2 + 3x + 1$$

$$(g \circ f)(x) = 3x^2 + 3x + 1$$

16) Find the distance between the points (6, 3) and (4, 1)

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ d &= \sqrt{(4-6)^2 + (1-3)^2} \\ d &\approx \sqrt{4+4} \\ d &\approx \sqrt{8} \\ d &\approx 2\sqrt{2} \end{aligned}$$

17) Write the equation of a circle in standard form if the endpoints of the diameter of the circle are

(4, 6) and (-6, -2)

$$\begin{aligned} d &= \sqrt{(-6-4)^2 + (-2-6)^2} \\ &\approx \sqrt{(-10)^2 + (-8)^2} \\ &\approx \sqrt{100+64} \\ &\approx \sqrt{164} \\ &\approx \sqrt{4 \cdot 41} \quad 2\sqrt{41} \end{aligned}$$

$$\text{radius} = \sqrt{41}$$

$$\text{center} = \left( \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2} \right)$$

$$\begin{aligned} &= \left( \frac{4+6}{2}, \frac{-2+6}{2} \right) \\ &\approx \left( \frac{10}{2}, \frac{4}{2} \right) \\ &\approx (5, 4) \end{aligned}$$

$$(x-5)^2 + (y-4)^2 = 41$$

18) If the given sequence is arithmetic, find the sum of the first 30 terms. If the sequence is geometric, find the sum of the first 8 terms.

a) 3, 9, 27, 81, ...

$$\frac{9}{3} = 3$$

$$\frac{27}{3} = 3$$

$$r = 3$$

geometric

$$\begin{aligned} S_8 &= \frac{a_1(1-r^8)}{1-r} \\ &= \frac{3(1-3^8)}{1-3} \\ &= \frac{3(1-6561)}{-2} \\ &\approx \frac{3(-6560)}{-2} \\ &= \frac{-19680}{-2} \\ S_8 &= 9840 \end{aligned}$$

b) 2, 7, 12, 17, ...

$$\begin{aligned} 2 &\sim 5 \\ 12 &\sim 7 \sim 5 \\ d &\sim 5 \\ &\sim 2 + (30-1)5 \\ &\sim 2 + (29)5 \\ &\sim 2 + 145 \\ a_{30} &\sim 147 \end{aligned}$$

$$S_{30} \sim a_1 + (n-1)d$$

$$\begin{aligned} &\sim 2 + (30-1)5 \\ &\sim 2 + (29)5 \\ &\sim 2 + 145 \\ S_{30} &\sim 147 \end{aligned}$$