Chapter 9, Section 1

Exponential Functions

Definition of an Exponential Function

The exponential function 'f' with base 'b' is defined by

$f(x) = b^x$ or $y = b^x$

Where b is a positive constant other than 1 ($b > 0$ and $b \neq 1$) and x is any real number.

Examples:

$f(x) = 2^x$ $g(x) = 10^x$ $h(x) = 3^{x+1}$ $f(x) = \left(\frac{1}{2}\right)^{x-1}$

Notice that the exponent is a variable.

Following functions are not exponential functions.

$F(x) = x^2$ $g(x) = (-1)^x$

Evaluate an Exponential Function

The exponential function $f(x) = 42.2(1.56)^x$ models the average amount spent, f(x)in dollars, at a shopping mall after x hours. What is the average amount spent, to the nearest dollar, after four hours?

Solution:

Since interested in amount spent after four hours, $x = 4$

Thus $f(4) = 42.2(1.56)^4$

Use a calculator: $f(4) = 250$

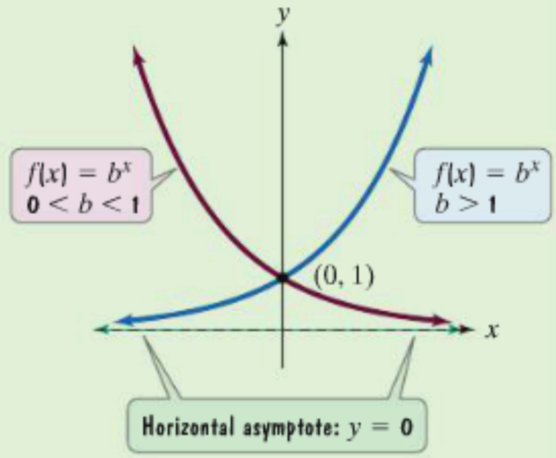
Graph $f(x) = 2^x$

Solution:

Set up a table of values then plot the points.

Characteristics of Exponential Functions of the Form $f(x) = b^x$

1. The domain of $f(x) = b^x$ consists of all real numbers: $(-\infty, \infty)$. The range of $f(x) = b^x$ consists of all positive real numbers: $(0, \infty)$.
2. The graphs of all exponential functions of the form $f(x) = b^x$ pass through the point $(0, 1)$ because $f(0) = b^0 = 1$ $(b \neq 0)$. The y-intercept is 1.
3. If $b > 1$, $f(x) = b^x$ has a graph that goes up to the right and is an increasing function. The greater the value of b, the steeper the increase.
4. If $0 < b < 1$, $f(x) = b^x$ has a graph that goes down to the right and is a decreasing function. The smaller the value of b, the steeper the decrease.
5. The graph of $f(x) = b^x$ approaches, but does not touch, the x-axis. The x-axis, or $y = 0$, is a horizontal asymptote.



Graph f(x) = 3^x

Graph g(x) = 3^{x-1}

Natural Base e

e irration number asspoximately 2.72, natural base.

Function f(x) = e^x, natural exponential function

Compound Interest.

Compounded Annually: $A = P(1+r)^t$

Where A, amount of money, P principal what will be worth after t years at interest rate, r

Compounded semiannually – interest paid twice a year, every six months

Compounded quartrly -interest paid four times a year, every three months

General, compound interest is paid n times a year, n compounding periods per year

So formula adjucted to account the number of compounding periods in a year, n

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

Coninuous compounding – number of compounding periods increases infinitely.

$$A = Pe^{rt}$$

Example:

Invest $8000 for 6 years. Choose between two accounts, 7% per year, compounded monthly or 6.85% per year, compounded continuously.