Chapter 9 Section 5
Exponential and Logarithmic Equations

Exponential Equations -equation containing a variable in an exponent.

Example:

$4^{x}=15$ $2^{3x-8}=16$ $40e^{0.6x}=240$

All exponential functions are one-to-one.

Solving Exponential Equations by Expressing Each Side as a Power of the Same Base.

If $b^{M}=b^{N}$, then $M=N$.

If the bases are the same, then the exponents are equal.

Example:

Solve the Exponential Equations

a) $2^{3x-8}=16$ b) $16^{x}=64$

Solution

Since 16 = 2^4 and 64 = 2^6 or 16 = 4^2 and 64 = 4^3

Rewrite each equation with this information

$2^{3x-8}=2^{4}$ $2^{4x}=2^{6}$ $4^{2x}=4^{3}$

Since the bases are the same, the exponents are equal so

$3x - 8 = 4$ $4x = 6$ $2x = 3$

Solve the equation

Try:

a) $5^{3x-6}=125$ b) $4^{x}=32$

Most exponential equations cannot be rewritten so that each side has the same base, so another way to solve these equations exist.

Use Logarithms to Solve Exponential Equations

1) Isolate the exponential expression
2) Take the common logarithm or natural logarithm on both sides of the equation
3) Simplify
4) Solve for the variable.

Example:
Solve the Exponential Equations

a) $4^x = 15$

b) $10^x = 120000$

Solution
Pick which base for the logarithm that one would like to use to solve these equations.

For a) use natural logarithm

$4^x = 15$

$\ln 4^x = \ln 15$

Use the power rule
x ln 4 = ln 15

Solve for x

$$x = \frac{\ln 15}{\ln 4}$$

Exact value

b) use common logarithm. Notice the 10

$10^x = 120000$

$\log 10^x = \log 120000$

x log 10 = log 120 000 or $\log 10^x = x$

since log 10 = 1

x = log 120 000

For approximate values, use your calculator.

Try:

a) $5^x = 134$

b) $10^x = 8000$

Try:

$40e^{0.6x} - 3 = 237$

Solution:
Isolate the exponential expression: $e^{0.6x}$
Then take the natural logarithm of both sides.

Another way to solve:

$4^x = 15$

Use the definition of the logarithm: $b^x = M$ is the same as $\log_b M = x$ so

$4^x = 15$ becomes $\log_4 15 = x$

Use the change of base rule

$$x = \frac{\log 15}{\log 4} \text{ or } x = \frac{\ln 15}{\ln 4}$$

The base of the logarithm depends.

Logarithmic Equation – equation containing a variable in a logarithmic expression.

Example:

$\log_4(x+3)=2$ $\ln(x+2)-\ln(4x+3)=\ln\left(\frac{1}{x}\right)$

Using Exponential Form to Solve Logarithmic Equations

1) Express the equation in the form: $\log_b M=c$

2) Use the definition of a logarithm to rewrite the equation in exponential form:
$\log_b M=x$ means $b^x=M$

3) solve for the variable.

4) Check the proposed solution in the original equation. M > 0.

Example:

a) $\log_4(x+3)=2$ b) $3\ln(2x)=12$

Solution

Rewrite so that the log does not have a coefficient and is isolated.

$\ln(2x)=4$

Rewrite in exponential form.

$4^2=x+3$ $e^4=2x$ Remember what the base is on ln

Solve for the variable, x

Try:

a) $\log_2(x-4)=3$ b) $4\ln(3x)=8$

Solve: $\log_2 x+\log_2(x-7)=3$ $\ln(x+2)-\ln(4x+3)=\ln\left(\frac{1}{x}\right)$