Chapter 9 Section 3
Logarithmic Functions

No horizontal line can be drawn that intersects the graph of an exponential function at more than one point. This means that the exponential functions in one-to-one and has an inverse.

Exponential function: f(x) = b^x
Find the inverse.

Do not have a method for solving $b^y = x$ for y.
Define a function, called the logarithmic function to solve this equation for y.

The inverse function of the exponential function with base 'b' is called the logarithmic function with base 'b'.

Definition of the Logarithmic Function:
For x > 0 and b > 0, b ≠ 1
y = $\log_b x$
is equivalent to $b^y = x$

y = $\log_b x$ and $b^y = x$ are different ways of expressing the same thing.
First equation: logarithmic form
Second equation: exponential form.

Change to Exponential form:
a) $2 = \log_5 x$ b) $\log_b 64 = 3$ c) $\log_3 7 = y$

Change to logarithmic form.
a) $12^2 = x$ b) $b^3 = 8$ c) $e^y = 9$

Evaluation of Logarithms
a) $\log_2 16$ b) $\log_3 9$ c) $\log_{25} 5$

Solution:
Rewrite in exponential form and observe.

Basic Properties:

$\log_b b = 1$ rewrite in exponential form: $b^1 = b$

$\log_b 1 = 0$ rewrite in exponential form: $b^0 = 1$

Check
Evaluate
a) $\log_7 7$ b) $\log_5 1$

Now, we can finish finding the inverse of f(x) = b^x
Step 1: Replace the f(x) with y: $y = b^x$

Step 2: Interchange the x and y: $x = b^y$

Step 3: Solve for y: $y = \log_b x$

Step 4: Replace y with $f^{-1}(x)$: $f^{-1}(x) = \log_b x$

The inverse of an exponential functions is the logarithmic function with the same base.

Inverse Properties of Logarithms
For b > 0 and b ≠ 1,

$\log_b b^x = x$

$b^{\log_b x} = x$

Check:
Evaluate:
a) $\log_4 4^5$ b) $6^{\log_6 9}$

Graph of Exponential and Logarithmic Function
Graph: $f(x) = 2^x$ and g(x) = $\log_2 x$

Solution:
What do you notice about the two functions?

Characteristics of Logarithmic Functions of the Form $f(x) = \log_b x$

1. The domain of $f(x) = \log_b x$ consists of all positive real numbers: $(0, \infty)$. The range of $f(x) = \log_b x$ consists of all real numbers: $(-\infty, \infty)$.
2. The graphs of all logarithmic functions of the form $f(x) = \log_b x$ pass through the point (1,0) because $f(1) = \log_b 1 = 0$. The x-intercept is 1. There is no y-intercept.
3. If $b > 1$, $f(x) = \log_b x$ has a graph that goes up to the right and is an increasing function.
4. If $0 < b < 1$, $f(x) = \log_b x$ has a graph that goes down to the right and is a decreasing function.
5. The graph of $f(x) = \log_b x$ approaches, but does not touch, the y-axis. The y-axis, or $x = 0$, is a vertical asymptote.

Domain of a Logarithmic Function
Domain of $f(x) = \log_b g(x)$ consists of all x for which g(x) > 0

Example
Domain of g(x) = $\log_4(x+3)$ is (-3. ∞)

Common Logarithms
Logarithms functions with the base 10 - common logarithmic function.
Function $f(x) = \log_{10} x$ is usually expressed as f(x) = log x.

The calculator with the LOG key can be used to evaluate common logarithms.

Evaluate:

a) log 1000 b) log $\frac{5}{2}$ c) $\frac{\log 5}{\log 2}$ d) log (- 3)

Properties of Common Logarithms

General Properties		Common Logarithms
1. $\log_b 1 = 0$		1. $\log 1 = 0$
2. $\log_b b = 1$		2. $\log 10 = 1$
3. $\log_b b^x = x$	Inverse properties	3. $\log 10^x = x$
4. $b^{\log_b x} = x$		4. $10^{\log x} = x$

Natural Logarithms
Logarithmic function with base e - natural logarithmic function
Function f(x) = $\log_e x$ is usually expressed as f(x) = ln x, read 'el en of x'.
The calculator with the LN key can be used to evaluate natural logarithms.

Evaluate:

a) ln 5 b) $\frac{\ln 12}{\ln 5}$ c) ln 2.58

The properties and domain for the natural logarithms and logarithmic functions are the same.

Properties of Natural Logarithms

General Properties		Natural Logarithms
1. $\log_b 1 = 0$		1. $\ln 1 = 0$
2. $\log_b b = 1$		2. $\ln e = 1$
3. $\log_b b^x = x$	Inverse properties	3. $\ln e^x = x$
4. $b^{\log_b x} = x$		4. $e^{\ln x} = x$