

Chapter 7 Section 6 Radical Equations

A radical equation is an equation in which the variable occurs in a square root, cube root, or any higher root.

Variable(s) occur in radicand(s)

Example:

$$\sqrt{2x+3}=5$$

$$\sqrt{3x+1}-\sqrt{x+4}=1$$

$$\sqrt[3]{3x-1}+4=0$$

Solving Radical Equations

$$\sqrt{x}=9$$

$$(\sqrt{x})^2=9^2$$

$$x=81$$

We solve radical equations with n th roots by raising both sides of the equation to the n th power.

If n is even, the solutions may not be solutions of the original equation. Always check the proposed solutions with the original equation.

Solutions that are not solutions of the given equation are called **extraneous solutions**.

Solve:

$$\sqrt{x-3}+6=5$$

Solution: Isolate a radical on one side.

Be sure to check the solution

Try:

a) $\sqrt{5x-1}=8$

b) $\sqrt{3x-2}-5=0$

Solve:

$$x + \sqrt{26 - 11x} = 4$$

Solution

$$\sqrt{26 - 11x} = 4 - x$$

Isolate a radical on one side.

$$\left(\sqrt{26 - 11x}\right)^2 = (4 - x)^2$$

Square both sides

$$26 - 11x = 18 - 8x + x^2$$

Simplify

$$0 = x^2 + 3x - 10$$

$$x = -5 \text{ or } x = 2$$

Solve the equation

Check solution

Try:

$$\text{a) } \sqrt{2x+1} = x-7$$

$$\text{b) } 3x - \sqrt{3x+7} = -5$$

Solving Equations That Have Two Radicals

Solve:

$$\sqrt{3x+1} - \sqrt{x+4} = 1$$

Solution

$$\sqrt{3x+1} - \sqrt{x+4} = 1$$

$$\sqrt{3x+1} = \sqrt{x+4} + 1$$

$$\left(\sqrt{3x+1}\right)^2 = \left(\sqrt{x+4} + 1\right)^2$$

$$3x+1 = x+4 + 2\sqrt{x+4} + 1$$

$$3x+1 = x+5 + 2\sqrt{x+4}$$

$$2x-4 = 2\sqrt{x+4}$$

$$(2x-4)^2 = (2\sqrt{x+4})^2$$

$$4x^2 - 16x + 16 = 4x + 16$$

$$4x^2 - 20x = 0$$

$$x = 0 \text{ or } x = 5$$

Check solutions

Try:

a) $\sqrt{x+5} - \sqrt{x-3} = 2$

b) $\sqrt{6x+2} = \sqrt{5x+3}$

c) $(3x-6)^{\frac{1}{3}} + 5 = 8$

Isolate a radical on one side.

Square both sides

Simplify

Combine like terms

Isolate a radical on one side.

Square both sides.

Note: Before one does this step. Could you have done something else?

Simplify

Solve the quadratic equation

Possible solutions.

Solving a Radical Equation

Solve:

$$(3x-1)^{\frac{1}{3}}+4=0$$

Solution:

$$(3x-1)^{\frac{1}{3}}+4=0$$

One can rewrite in radical form, but it is not necessary.

$$\sqrt[3]{3x-1}+4=0$$

$$(3x-1)^{\frac{1}{3}}=-4$$

Isolate the radical term, term with the rational exponent

$$\left[(3x-1)^{\frac{1}{3}}\right]^3=(-4)^3$$

Cube both sides

$$3x-1=-64$$

Simplify

$$x=-21$$

Solve the equation

Try:

$$\text{a) } (x-3)^{\frac{1}{2}}+8=6$$

$$\text{b) } 3x^{\frac{1}{3}}=(x^2+17x)^{\frac{1}{3}}$$

Extra:

Solve the formula for the specified variable.

- Solve for V: $r = \sqrt{\frac{3V}{\pi h}}$

- Solve for m: $v = \sqrt{\frac{FR}{m}}$