Chapter 7 Section 2
Rational Exponents

Exponents that are rational - fractions

Example:

$c^{\frac{1}{2}}$ $a^{\frac{2}{3}}$

The Definition of $a^{\frac{1}{n}}$

If $\sqrt[n]{a}$ represents a real number and $n \geq 2$ is an integer, then

$$a^{\frac{1}{n}} = \sqrt[n]{a}.$$

The denominator of the rational exponent is the radical's index.

If n is even, a must be nonnegative. If n is odd, a can be any real number.

Notice: the denominator in the exponent and the numerator is the index of the radical.

Using the definition

Use radical notation to rewrite the expression. Simplify, if possible.

a) $64^{\frac{1}{2}}$

b) $(-125)^{\frac{1}{3}}$

Solution:

$64^{\frac{1}{2}}$

$\sqrt{64}$

8

$(-125)^{\frac{1}{3}}$

$\sqrt[3]{-125}$

- 5

Try:

a) $25^{\frac{1}{2}}$

b) $(-8)^{\frac{1}{3}}$

c) $(5xy^2)^{\frac{1}{4}}$

Rewrite with rational exponents

a) $\sqrt[5]{13ac}$

b) $\sqrt[7]{\dfrac{xy^2}{17}}$

Solution

a) $\sqrt[5]{13ac}$

$(13ac)^{\frac{1}{5}}$

b) $\sqrt[7]{\dfrac{xy^2}{17}}$

$\left(\dfrac{xy^2}{17}\right)^{\frac{1}{7}}$

Try

a) $\sqrt[5]{5xy}$

b) $\sqrt[5]{\dfrac{a^2b}{2}}$

The Definition of $a^{\frac{m}{n}}$

If $\sqrt[n]{a}$ represents a real number, $\dfrac{m}{n}$ is a positive rational number reduced to lowest terms, and $n \geq 2$ is an integer, then

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m.$$

First take the nth root of a.

and

$$a^{\frac{m}{n}} = \sqrt[n]{a^m}.$$

First raise a to the m power.

The preferred way is to take the root first, because smaller numbers are involved.
Notice that the rational exponent consists of two parts. The numerator is the exponent and the denominator is the radical's index.

Using the definition:

Use radical notation to rewrite and simplify:

a) $(1000)^{\frac{2}{3}}$

b) $16^{\frac{2}{3}}$

c) $-32^{\frac{3}{5}}$

Solution:

a)

$(1000)^{\frac{2}{3}}$

$\left(\sqrt[3]{1000}\right)^2$

$\left(\sqrt[3]{10^3}\right)^2$

10^2

100

b)

$\left(\sqrt{16}\right)^3$

4^3

64

c)

$-\left(\sqrt[5]{32}\right)^3$

-2^3

-8

Try:

Use radical notation to rewrite and simplify:

c) $8^{\frac{4}{3}}$

d) $-81^{\frac{3}{4}}$

Rewrite with rational exponents:

e) $\sqrt[3]{7^5}$

f) $\left(\sqrt[5]{2xy}\right)^7$

The Definition of $a^{-\frac{m}{n}}$

If $a^{\frac{m}{n}}$ is a nonzero real number, then

$$a^{-\frac{m}{n}} = \frac{1}{a^{\frac{m}{n}}}.$$

Using the definition

Rewrite the expression with a positive exponent then simplify, if possible

a) $36^{-\frac{1}{2}}$ b) $(7xy)^{-\frac{4}{7}}$

solution:

$36^{-\frac{1}{2}}$

$\dfrac{1}{36^{\frac{1}{2}}}$

$\dfrac{1}{\sqrt{36}}$

$\dfrac{1}{6}$

$(7xy)^{-\frac{4}{7}}$

$\dfrac{1}{(7xy)^{\frac{4}{7}}}$

The properties of Rational Exponents are the same for integer exponents.

Properties of Rational Exponents

If m and n are rational exponents, and a and b are real numbers for which the following expressions are defined, then

1. $b^m \cdot b^n = b^{m+n}$ When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.
2. $\dfrac{b^m}{b^n} = b^{m-n}$ When dividing exponential expressions with the same base, subtract the exponents. Use this difference as the exponent of the common base.
3. $(b^m)^n = b^{mn}$ When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.
4. $(ab)^n = a^n b^n$ When a product is raised to a power, raise each factor to that power and multiply.
5. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$ When a quotient is raised to a power, raise the numerator to that power and divide by the denominator to that power.

Rational exponents are simplified when

a) No parentheses appear
b) No powers are raised to powers
c) Each base occurs only once
d) No negative or zero exponents appear.

Use the properties and simplify.

g) $5^{\frac{1}{7}} \bullet 5^{\frac{6}{7}}$

h) $\dfrac{32x^{\frac{1}{2}}}{16x^{\frac{3}{4}}}$

i) $\left(x^{-\frac{2}{5}}y^{\frac{1}{3}}\right)^{\frac{1}{2}}$

Use rational exponents to simplify

a) $\sqrt[10]{x^5}$

b) $\sqrt[3]{27x^{15}}$

c) $\sqrt[3]{\sqrt{x}}$