Chapter 6 section 6
Rational Equations

Rational Equations, fractional equations – equation containing one or more rational expressions.

Example:

$$80 = \frac{120x}{100 - x}$$

The characteristic of many rational equations, is that there is a variable in the denominator.

Solving Rational Equations:
Use the LCD and multiply each term of the equation. This will clear an equation of fractions.
Or
Add or subtract the fractional expression then solve the equation.

Example: Solve the equation

$$\frac{x+4}{2x} + \frac{x+20}{3x} = 3, \ x \neq 0$$

Solution:
The LCD is 6x, so multiply each term by 6x

$$\left(\frac{x+4}{2x}\right)\left(\frac{6x}{1}\right) + \left(\frac{x+20}{3x}\right)\left(\frac{6x}{1}\right) = 3(6x)$$

Reduce, divide out the common factor.

$$(x+4)(3) + (x+20)(2) = 18x$$

Use the distributive property and combine similar terms

$$3x + 12 + 2x + 40 = 18x$$
$$5x + 52 = 18x$$

Solve the equation
$$x = 4$$

Check
Since the solution, 4, is not part of the restriction x ≠ 0, it should check.

Add or subtract the fractional expression then solve the equation.

$$\frac{x+4}{2x} + \frac{x+20}{3x} = 3, \ x \neq 0$$

Common denominator is 6x, so write an equivalent fraction with the denominator 6x

$$\frac{3(x+4)}{6x} + \frac{2(x+20)}{6x} = 3 \qquad\qquad \frac{3(x+4)}{6x} + \frac{2(x+20)}{6x} = \frac{18x}{6x}$$

Simplify the numerators.

$$\frac{3x+12}{6x} + \frac{2x+40}{6x} = 3 \qquad\qquad \frac{3x+12}{6x} + \frac{2x+40}{6x} = \frac{18x}{6x}$$

Add the fractions

$$\frac{5x+52}{6x} = 3 \qquad\qquad \frac{5x+52}{6x} = \frac{18x}{6x}$$

Solve the equation

5x + 52 = 18x

x = 4

Try:

a) $\dfrac{2x}{x-3} + \dfrac{6}{x+3} = \dfrac{28}{(x+3)(x-3)}$

b) $\dfrac{9}{4x} - \dfrac{5}{2x} = \dfrac{3}{4}$

Simplifying a Rational Expression.
The above instruction goes with which problems. Why?

Problem 1   Problem 2

$$\frac{x+1}{x+10} = \frac{x-2}{x+4} \qquad\qquad\qquad \frac{x+6}{2x} + \frac{x-24}{5x}$$