

Chapter 6 section 2
Adding and Subtracting Rational Expressions

Addition, subtracting – denominators are the same.

Add or subtract the numerators and copy the denominator, then reduce.

Example 1

$$\frac{2}{9} + \frac{5}{9}$$
$$\frac{2+5}{9}$$
$$\frac{7}{9}$$

Example 2

$$\frac{3y^2}{x^2} - \frac{4y}{x^2}$$
$$\frac{3y^2 - 4y}{x^2}$$

Try these problems

a)

$$\frac{x}{x-3} + \frac{11x+5}{x-3}$$

b)

$$\frac{3x}{7x-4} - \frac{2x-1}{7x-4}$$

c)

$$\frac{2x^3 - 3y^3}{x^2 - y^2} - \frac{x^3 - 2y^3}{x^2 - y^2}$$

Addition, Subtraction – Denominators are Different

Find the least common denominator: LCD

Finding the Least Common Denominator

1. Factor each denominator completely.
2. List the factors of the first denominator.
3. Add to the list in step 2 any factors of the second denominator that do not appear in the list.
4. Form the product of each different factor from the list in step 3. This product is the least common denominator.

Find the LCD

$$\frac{9}{7x^2 + 28x} \text{ and } \frac{11}{x^2 + 8x + 16}$$

1) Factor each denominator

$$\begin{array}{ll} 7x^2 + 28x & x^2 + 8x + 16 \\ 7x(x + 4) & (x + 4)^2 \end{array}$$

2) List the factors of the first denominator

$$7x(x + 4)$$

3) Add any unlisted factors from the second denominator

$$7x(x + 4), (x + 4)$$

4) Product of the factors in #3. This is the Least Common Denominator

$$7x(x + 4)^2$$

Find the least common denominator

d)

$$\frac{11}{25x^2} \text{ and } \frac{14}{35x}$$

e)

$$\frac{7}{y^2 - 4} \text{ and } \frac{15}{y(y + 2)}$$

f)

$$\frac{7}{y^2 - 5y - 6} \text{ and } \frac{y}{y^2 - 4y - 5}$$

Write fraction with the LCD

After the LCD is found, an equivalent fraction is written with the LCD.

Fraction:

$$\frac{3}{10x^2}$$

common denominator: $30x^2$

multiply by a form of 'one' to get the LCD

$$\frac{3}{10x^2} \cdot \frac{3}{3}$$

$$\frac{9}{30x^2}$$

Once each fraction is written with the same denominator, the addition or subtraction can be done.

Adding and Subtracting Rational Expressions That Have Different Denominators

1. Find the LCD of the rational expressions.
2. Rewrite each rational expression as an equivalent expression whose denominator is the LCD. To do so, multiply the numerator and the denominator of each rational expression by any factor(s) needed to convert the denominator into the LCD.
3. Add or subtract numerators, placing the resulting expression over the LCD.
4. If possible, simplify the resulting rational expression.

Example:

$$\frac{x-1}{x^2+x-6} - \frac{x-2}{x^2+4x+3}$$

Solution:

1) Find the LCD

Factor each denominator

$$\begin{aligned} x^2 + x - 6 \\ (x + 3)(x - 2) \end{aligned}$$

$$\begin{aligned} x^2 + 4x + 3 \\ (x + 3)(x + 1) \end{aligned}$$

List the factors of the first: $(x + 3)(x - 2)$

Add the factors of the second that is not in the first: $(x + 1)$

Thus, the LCD is:

$$(x + 3)(x - 2)(x + 1)$$

2) Rewrite each fraction with the LCD

$$\frac{x-1}{x^2+x-6}$$

$$\frac{x-2}{x^2+4x+3}$$

$$\frac{x-1}{(x+3)(x-2)}$$

$$\frac{x-2}{(x+3)(x+1)}$$

$$\frac{(x-1)(x+1)}{(x+3)(x-2)(x+1)}$$

$$\frac{(x-2)(x-2)}{(x+3)(x-2)(x+1)}$$

Once the fractions are written with the LCD, one can do the operation

3)

$$\frac{(x-1)(x+1)}{(x+3)(x-2)(x+1)} - \frac{(x-2)(x-2)}{(x+3)(x-2)(x+1)}$$

4) Multiply the numerators and subtract

$$\frac{x^2-1}{(x+3)(x-2)(x+1)} - \frac{x^2-4x+4}{(x+3)(x-2)(x+1)}$$

$$\frac{x^2 - 1 - x^2 + 4x + 4}{(x + 3)(x - 2)(x + 1)}$$

5) Answer:

$$\frac{4x + 3}{(x + 3)(x - 2)(x + 1)}$$

Try:

g)

$$\frac{x-2}{x+7} + \frac{x+7}{x-2}$$

h)

$$\frac{x+2y}{x^2+4xy+4y^2} - \frac{2x}{x^2-4y^2}$$