Chapter 4 section 3
Notes – Blitzer 7th

Equations and Inequalities Involving Absolute Value

Absolute Value of ‘a’ denoted by | a | is the distance from 0 to ‘a’ on the number line.

Rewriting an Absolute Value Equation Without Absolute Value Bars

If c is a positive real number and u represents any algebraic expression, then $|u| = c$ is equivalent to $u = c$ or $u = -c$.

Example 1: page 276:
Solving an Equation Involving Absolute Value.

Solve: |2x – 3| = 11

Solution:
* Rewrite the equation without the absolute value bars.

Hint: Cover 2x – 3 and ask what value goes there? 11 or -11. So…
2x – 3 = 11 or 2x – 3 = -11

* Solve the equations
X = 7 x = - 4

Check.

Try:
Example 2: Page 276
Solve: 5 |1 – 4x | = 0

How about: 5 |1 – 4x | - 15 = 0

What about: |3x – 1 | = |x + 5 |

Guidelines for solution.
1) Rewrite without the absolute value bars. Be sure that there are two equations.
2) Solve each equation.
3) Check.

Inequalities Involving Absolute Value

|x| < 2 distance from 0 to x on the number line that is less than 2.

Solving Absolute Value Inequalities of the Form $|u| < c$

If c is a positive real number and u represents any algebraic expression, then

$$|u| < c \text{ is equivalent to } -c < u < c.$$

This rule is valid if $<$ is replaced by $\leq$.

Solving an Absolute Value Inequality of the Form |u| < c
Example 4: page 278:

Solve and graph the solution set on the number line:

$|x - 4| < 3$

Solution:

1) Rewrite without the absolute value bars:

$-3 < x - 4 < 3$

2) Solve the compound inequality

$1 < x < 7$

1) Graph the solution

Try: $-2|x+5| + 7 \geq -13$

To solve, isolate the absolute value expression. Get only the expression on the left side of the equation.

Try 2: $|3x + 5| < 17$

Absolute value inequalities of the form: | u | > c

|x | > 2 Distance from 0 to x on the number line that is greater than 2.

Solving Absolute Value Inequalities of the Form $|u| > c$

If c is a positive real number and u represents any algebraic expression, then

$$|u| > c \text{ is equivalent to } u < -c \text{ or } u > c.$$

This rule is valid if $>$ is replaced by $\geq$.

Example 6: page 280:
Solve and graph the solution set on the number line.

$|2x + 3| \ge 5$

Solution:

1) Rewrite without the absolute value bars
$2x + 3 \le -5$ or $2x + 3 \ge 5$

2) Solve the inequalities

3) Graph the solution set.

Try: $|2x - 5| > 3$

Try 2: $|5x - 2| > 13$

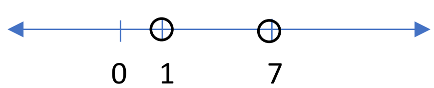
Another way to solve Absolute Value Inequalities.

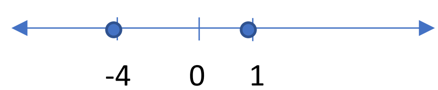
$|x - 4| < 3$ $|2x + 3| \ge 5$

1) Replace the inequality symbol with an equal sign
$|x - 4| = 3$ $|2x + 3| = 5$

2) Solve the equation.
$x - 4 = 3$ or $x - 4 = -3$ $2x + 3 = 5$ or $2x + 3 = -5$
$x = 7$ or $x = 1$ $x = 1$ or $x = -4$

3) Graph these points on the number line. Note the original inequality.





4) Notice on both graphs, there are 3 regions. Pick a point in each region as a test point and put into the original inequality. If the inequality become true, one shades that region. If false, then one does not shade.

5) Take x = 0. Do the substitution, $|x - 4| < 3$ $|2x + 3| \ge 5$
$|0 - 4| < 3$ $|2(0) + 3| \ge 5$
$|-4| < 3$ $|0 + 3| \ge 5$
$4 < 3$ false $3 \ge 5$ false
Do not shade the region that has 0

6) Tries the other regions. The regions that are true, is the solution and shading occurs.

Try: $-2|5 - x| < -6$ $3|2x - 1| + 2 > 8$

Summary:
If $c > 0$

- $|u| = c$ is equivalent to $u = c$ or $u = -c$
- $|u| < c$ is equivalent to $-c < u < c$
- $|u| > c$ is equivalent to $u < -c$ or $u > c$