## SECTION 8.1 PROBLEM SET: SAMPLE SPACES AND PROBABILITY

In problems 1 - 6, write a sample space for the given experiment.

|  |  |
| --- | --- |
| 1) A die is rolled. | 2) A penny and a nickel are tossed. |
| 3) A die is rolled, and a coin is tossed. | 4) Three coins are tossed. |
| 5) Two dice are rolled. | 6) A jar contains four marbles numbered 1, 2, 3, and 4. Two marbles are drawn. |

In problems 7 - 12, one card is randomly selected from a deck. Find the following probabilities.

|  |  |
| --- | --- |
| 7) P( an ace) | 8) P( a red card) |
| 9) P( a club) | 10) P( a face card) |
| 11) P(a jack or a spade) | 12) P(a jack and a spade) |

***SECTION 8.1 PROBLEM SET: SAMPLE SPACES AND PROBABILITY***

For problems 13 – 16: A jar contains 6 red, 7 white, and 7 blue marbles. If one marble is chosen at random, find the following probabilities.

|  |  |
| --- | --- |
| 13) P(red) | 14) P(white) |
| 15) P(red or blue) | 16) P(red and blue) |

For problems 17 – 22: Consider a family of three children. Find the following probabilities.

|  |  |
| --- | --- |
| 17) P(two boys and a girl) | 18) P(at least one boy) |
| 19) P(children of both sexes) | 20) P(at most one girl) |
| 21) P(first and third children are male) | 22) P(all children are of the same gender) |

***SECTION 8.1 PROBLEM SET: SAMPLE SPACES AND PROBABILITY***

For problems 23 – 27: Two dice are rolled. Find the following probabilities.

|  |  |
| --- | --- |
| 23) P(the sum of the dice is 5) | 24) P(the sum of the dice is 8) |
| 25) P(the sum is 3 or 6) | 26) P(the sum is more than 10) |
| 27) P(the result is a double) (Hint: a double means that both dice show the same value) | |

For problems 28-31: A jar contains four marbles numbered 1, 2, 3, and 4. Two marbles are drawn randomly WITHOUT REPLACEMENT. That means that after a marble is drawn it is NOT replaced in the jar before the second marble is selected. Find the following probabilities.

|  |  |
| --- | --- |
| 28) P(the sum of the numbers is 5) | 29) P(the sum of the numbers is odd) |
| 30) P(the sum of the numbers is 9) | 31) P(one of the numbers is 3) |

For problems 32-33: A jar contains four marbles numbered 1, 2, 3, and 4. Two marbles are drawn randomly WITH REPLACEMENT. That means that after a marble is drawn it is replaced in the jar before the second marble is selected. Find the following probabilities.

|  |  |
| --- | --- |
| 32) P(the sum of the numbers is 5) | 33) P(the sum of the numbers is 2) |

## SECTION 8.2 PROBLEM SET: MUTUALLY EXCLUSIVE EVENTS AND THE ADDITION RULE

Determine whether the following pair of events are mutually exclusive.

|  |  |
| --- | --- |
| 1) A = {A person earns more than $25,000}  B = {A person earns less than $20,000} | 2) A card is drawn from a deck.  C = {It is a King} D = {It is a heart}. |
| 3) A die is rolled.  E = {An even number shows}  F = {A number greater than 3 shows} | 4) Two dice are rolled.  G = {The sum of dice is 8}  H = {One die shows a 6} |
| 5) Three coins are tossed.  I = {Two heads come up}  J = {At least one tail comes up} | 6) A family has three children.  K = {First born is a boy}  L = {The family has children of both sexes} |

Use the Addition Rule to find the following probabilities.

|  |  |
| --- | --- |
| 7) A card is drawn from a deck.  Events C and D are:  C = {It is a king} D = {It is a heart}  Find P(C or D). | 8) A die is rolled. The events E and F are:  E = {An even number shows}  F = {A number greater than 3 shows}  Find P(E or F). |
| 9) Two dice are rolled. Events G and H are:  G = {The sum of dice is 8}  H ={Exactly one die shows a 6}  Find P(G or H). | 10) Three coins are tossed> Events I and J are:  I = {Two heads come up}  J = {At least one tail comes up}  Find P(I or J). |

***SECTION 8.2 PROBLEM SET: MUTUALLY EXCLUSIVE EVENTS AND THE ADDITION RULE***

Use the Addition Rule to find the following probabilities.

|  |  |
| --- | --- |
| 11) At a college, 20% of the students take Finite Mathematics, 30% take Statistics and  10% take both. What percent of students take Finite Mathematics or Statistics? | 12) This quarter, there is a 50% chance that Jason will pass Accounting, a 60% chance that he will pass English, and 80% chance that he will pass  at least one of these two courses. What is the probability that he will pass both Accounting  and English? |

Questions 13 – 20 refer to the following: The table shows the distribution of Democratic and Republican U.S by gender in the 114th Congress as of January 2015.

|  |  |  |  |
| --- | --- | --- | --- |
|  | MALE(M) | FEMALE(F) | TOTAL |
| DEMOCRATS (D) | 30 | 14 | 44 |
| REPUBLICANS(R) | 48 | 6 | 54 |
| OTHER (T) | 2 | 0 | 2 |
| TOTALS | 80 | 20 | 100 |

Use this table to determine the following probabilities.

|  |  |
| --- | --- |
| 13) P(M and D) | 14) P(F and R) |
| 15) P(M or D) | 16) P(F or R) |
| 17) P(Mc or R) | 18) P(M or F) |
| 19) Are the events F, R mutually exclusive?  Use probabilities to support your conclusions. | 20) Are the events F, T mutually exclusive?  Use probabilities to support your conclusion. |

***SECTION 8.2 PROBLEM SET: MUTUALLY EXCLUSIVE EVENTS AND THE ADDITION RULE***

Use the Addition Rule to find the following probabilities.

|  |  |
| --- | --- |
| 21) If P(E) = .5 , P(F) = .4 , E and F are mutually exclusive, find P(E and F). | 22) If P(E) = .4 , P(F) = .2 , E and F are mutually exclusive, find P(E or F). |
| 23) If P(E) = .3, P(E or F) = .6 , P(E and F) = .2, find P(F). | 24) If P(E) = .4, P(F) = .5 , P(E or F) = .7, find P(E and F). |

|  |  |
| --- | --- |
| 25) In a box of assorted cookies, 36% of cookies contain chocolate and 12% of cookies contain nuts. 8% of cookies have both chocolats and nuts. Sean is allergic to chocolate and nuts. Find the probability that a cookie has chocolate chips **or** nuts (he can’t eat it). | 26) At a college, 72% of courses have final exams and 46% of courses require research papers.  32% of courses have both a research paper and a final exam. Let F be the event that a course has a final exam and R be the event that a course requires a research paper.  Find the probability that a course requires a final exam **or** a research paper. |

*Questions 25 and 26 are adapted from Introductory Statistics from OpenStax under a creative Commons Attribution 3.0 Unported License, available for download free athttp://cnx.org/content/col11562/latest u*

## SECTION 8.3 PROBLEM SET: PROBABILITIES USING TREE DIAGRAMS AND COMBINATIONS

Two apples are chosen from a basket containing five red and three yellow apples.   
Draw a tree diagram below, and find the following probabilities.

|  |  |
| --- | --- |
| 1) P( both red) | 2) P(one red, one yellow) |
| 3) P(both yellow) | 4) P(First red and second yellow) |

A basket contains six red and four blue marbles. Three marbles are drawn at random.   
Find the following probabilities using the method shown in Example 2. Do not use combinations.

|  |  |
| --- | --- |
| 5) P( All three red) | 6) P(two red, one blue) |
| 7) P(one red, two blue) | 8) P(first red, second blue, third red) |

Three marbles are drawn from a jar containing five red, four white, and three blue marbles.   
Find the following probabilities using combinations.

|  |  |
| --- | --- |
| 9) P(all three red) | 10) P(two white and 1 blue) |
| 11) P(none white) | 12) P(at least one red) |

***SECTION 8.3 PROBLEM SET: PROBABILITIES USING TREE DIAGRAMS AND COMBINATIONS***

A committee of four is selected from a total of 4 freshmen, 5 sophomores, and 6 juniors.   
Find the probabilities for the following events.

|  |  |
| --- | --- |
| 13) At least three freshmen. | 14) No sophomores. |
| 15) All four of the same class. | 16) Not all four from the same class. |
| 17) Exactly three of the same class. | 18) More juniors than freshmen and sophomores combined. |

Five cards are drawn from a deck. Find the probabilities for the following events.

|  |  |
| --- | --- |
| 19) Two hearts, two spades, and one club. | 20) A flush of any suit (*all cards of a single suit*). |
| 21) A full house of nines and tens (*3 nines and 2 tens*). | 22) Any full house. |
| 23) A pair of nines and a pair of tens (*and the fifth card is not a nine or ten*). | 24) Any two pairs (*two cards of one value, two more cards of another value, and the fifth card does not have the same value as either pair*). |

***SECTION 8.3 PROBLEM SET: PROBABILITIES USING TREE DIAGRAMS AND COMBINATIONS***

Jorge has 6 rock songs, 7 rap songs and 4 country songs that he likes to listen to while he exercises.   
He randomly selects six (6) of these songs to create a playlist to listen to today while he exercises.

Find the following probabilities:

|  |  |
| --- | --- |
| 25) P(playlist has 2 songs of each type) | 26) P(playlist has no country songs) |
| 27) P(playlist has 3 rock, 2 rap, and 1 country song) | 28) P(playlist has 3 or 4 rock songs and the rest are rap songs) |

A project is staffed 12 people: 5 engineers, 4 salespeople, and 3 customer service representatives.   
A committee of 5 people is selected to make a presentation to senior management.

Find the probabilities of the following events.

|  |  |
| --- | --- |
| 29) The committee has 2 engineers, 2 salespeople, and 1 customer service representative. | 30) The committee contains 3 engineer and 2 salespeople. |
| 31) The committee has no engineers. | 32) The committee has all salespeople. |

Do the following birthday problems.

|  |  |
| --- | --- |
| 33) If there are 5 people in a room, what is the probability that no two have the same birthday? | 34) If there are 5 people in a room, find the probability that at least 2 have the same birthday. |

## SECTION 8.4 PROBLEM SET: CONDITIONAL PROBABILITY

Questions 1 – 4: Do these problems using the conditional probability formula: P(A | B) = .

|  |  |
| --- | --- |
| 1) A card is drawn from a deck. Find the conditional probability of P(a queen | a face card). | 2) A card is drawn from a deck. Find the conditional probability of P(a queen | a club). |
| 3) A die is rolled. Find the conditional probability that it shows a three if it is known that an odd number has shown. | 4) If P(A) = .3 , P(B) = .4, P(A and B) = .12, find: a) P(A | B)  b) P(B | A) |

Questions 5 – 8 refer to the following: The table shows the distribution of Democratic and Republican U.S. Senators by gender in the 114th Congress as of January 2015.

|  |  |  |  |
| --- | --- | --- | --- |
|  | MALE(M) | FEMALE(F) | TOTAL |
| DEMOCRATS (D) | 30 | 14 | 44 |
| REPUBLICANS(R) | 48 | 6 | 54 |
| OTHER (T) | 2 | 0 | 2 |
| TOTALS | 80 | 20 | 100 |

Use this table to determine the following probabilities:

|  |  |
| --- | --- |
| 5) P(M | D) | 6) P(D | M) |
| 7) P(F | R) | 8) P(R | F) |

***SECTION 8.4 PROBLEM SET: CONDITIONAL PROBABILITY***

Do the following conditional probability problems.

|  |  |
| --- | --- |
| 9) At a college, 20% of the students take Finite Math, 30% take History, and 5% take both Finite Math and History. If a student is chosen at random, find the following conditional probabilities.  a) He is taking Finite Math given that he is taking History.  b) He is taking History assuming that he is taking Finite Math. | 10) At a college, 60% of the students pass Accounting, 70% pass English, and 30% pass both of these courses. If a student is selected at random, find the following conditional probabilities.  a) He passes Accounting given that he passed English.  b) He passes English assuming that he passed Accounting. |
| 11) If P(F) = .4 , P(E | F) = .3, find P(E and F). | 12) P(E) = .3, P(F) = .3; E and F are mutually exclusive. Find P(E | F). |
| 13) If P(E) = .6 , P(E and F) = .24, find P(F | E). | 14) If P(E and F) = .04 , P(E | F) = .1, find P(F). |

At a college, 72% of courses have final exams and 46% of courses require research papers.   
32% of courses have both a research paper and a final exam. Let F be the event that a course has a final exam and R be the event that a course requires a research paper.

|  |  |
| --- | --- |
| 15) Find the probability that a course has a final exam given that it has a research paper. | 16) Find the probability that a course has a research paper if it has a final exam. |

***SECTION 8.4 PROBLEM SET: CONDITIONAL PROBABILITY***

Consider a family of three children. Find the following probabilities.

|  |  |
| --- | --- |
| 17) P(two boys | first born is a boy) | 18) P(all girls | at least one girl is born) |
| 19) P(children of both sexes | first born is a boy) | 20) P(all boys | there are children of both sexes) |

Questions 21 – 26 refer to the following:   
The table shows highest attained educational status for a sample of US residents age 25 or over:  
based on data from http://www.census.gov/hhes/socdemo/education/data/cps/2010/Table1-01.xls

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | (D) Did not Complete  High School | (H) High School  Graduate | (C)  Some  College | (A) Associate  Degree | (B) Bachelor  Degree | (G)  Graduate  Degree | TOTAL |
| 25-44 (R) | 95 | 228 | 143 | 81 | 188 | 61 | 796 |
| 45-64 (S) | 83 | 256 | 136 | 80 | 150 | 67 | 772 |
| 65+ (T) | 96 | 191 | 84 | 36 | 80 | 41 | 528 |
| Total | 274 | 675 | 363 | 197 | 418 | 169 | 2096 |

Use this table to determine the following probabilities:

|  |  |  |
| --- | --- | --- |
| 21) P(C | T) | 22) P(S | A) | 23) P(C and T) |
| 24) P(R | B) | 25) P(B | R) | 26) P(G|S) |

## SECTION 8.5 PROBLEM SET: INDEPENDENT EVENTS

The distribution of the number of fiction and non-fiction books checked out at a city's main library and at a smaller branch on a given day is as follows.

|  |  |  |  |
| --- | --- | --- | --- |
|  | MAIN (M) | BRANCH (B) | TOTAL |
| FICTION (F) | 300 | 100 | 400 |
| NON-FICTION (N) | 150 | 50 | 200 |
| TOTALS | 450 | 150 | 600 |

Use this table to determine the following probabilities:

|  |  |
| --- | --- |
| 1) P(F) | 2) P(M | F) |
| 3) P(N | B) | 4) Is the fact that a person checks out a fiction book independent of the main library?  Use probabilities to justify your conclusion. |

For a two-child family, let the events E, F, and G be as follows.

E: The family has at least one boy   
F: The family has children of both sexes  
G: The family's first born is a boy

|  |  |
| --- | --- |
| 5) Find the following.  a) P(E)  b) P(F)  c) P(E  F)  d) Are E and F independent?   Use probabilities to justify your conclusion. | 6) Find the following.  a) P(F)    b) P(G)  c) P(F  G)    d) Are F and G independent?  Use probabilities to justify your conclusion. |

***SECTION 8.5 PROBLEM SET: INDEPENDENT EVENTS***

Do the following problems involving independence.

|  |  |
| --- | --- |
| 7) If P(E) = .6, P(F) = .2, and E and F are independent, find P(E and F). | 8) If P(E) = .6, P(F) = .2, and E and F are independent, find P(E or F). |
| 9) If P(E) = .9, P(F | E) = .36, and E and F are independent, find P(F). | 10) If P(E) = .6, P(E or F) = .8, and E and F are independent, find P(F). |
| 11) In a survey of 100 people, 40 were casual drinkers, and 60 did not drink. Of the ones who drank, 6 had minor headaches. Of the non-drinkers, 9 had minor headaches. Are the events "drinkers" and "had headaches" independent? | 12) It is known that 80% of the people wear seat belts, and 5% of the people quit smoking last year. If 4% of the people who wear seat belts quit smoking, are the events, wearing a seat belt and quitting smoking, independent? |

***SECTION 8.5 PROBLEM SET: INDEPENDENT EVENTS***

|  |  |
| --- | --- |
| 13) John's probability of passing statistics is 40%, and Linda's probability of passing the same course is 70%. If the two events are independent, find the following probabilities.  a) P( both of them will pass statistics)  b) P(at least one of them will pass statistics) | 14) Jane is flying home for the Christmas holidays. She has to change planes twice. There is an 80% chance that she will make the first connection, and a 90% chance that she will make the second connection. If the two events are independent, find the probabilities:  a) P( Jane will make both connections)  b) P(Jane will make at least one connection) |

For a three-child family, let the events E, F, and G be as follows.

E: The family has at least one boy   
F: The family has children of both sexes  
G: The family's first born is a boy

|  |  |
| --- | --- |
| 15) Find the following.  a) P(E)    b) P(F)  c) P(E  F)    d) Are E and F independent? | 16) Find the following.  a) P(F)    b) P(G)  c) P(F  G)    d) Are F and G independent? |

***SECTION 8.5 PROBLEM SET: INDEPENDENT EVENTS***

|  |  |
| --- | --- |
| 17) P(K|D) = 0.7, P(D) = 0.25 and P(K)=0.7  a. Are events K and D independent?  Use probabilities to justify your conclusion.  b. Find P(K D) | 18) P(R|S) = 0.4, P(S) = 0.2 and P(R)=0.3  a. Are events R and S independent?  Use probabilities to justify your conclusion.  b. Find P(R S) |
| 19) At a college:  54% of students are female 25% of students are majoring in engineering. 15% of female students are majoring in engineering.  Event E = student is majoring in engineering Event F = student is female  a. Are events E and F independent?  Use probabilities to justify your conclusion.  b. Find P(E F) | 20) At a college:  54% of all students are female  60% of all students receive financial aid.   60% of female students receive financial aid.  Event A = student receives financial aid Event F = student is female  a. Are events A and F independent?  Use probabilities to justify your conclusion.  b. Find P(A F) |

## SECTION 8.6 PROBLEM SET: CHAPTER REVIEW

1) Two dice are rolled. Find the probability that the sum of the dice is

a) four b) five

2) A jar contains 3 red, 4 white, and 5 blue marbles.   
If a marble is chosen at random, find the following probabilities:

a) P(red or blue) b) P(not blue)

3) A card is drawn from a standard deck. Find the following probabilities:

a) P(a jack or a king) b) P(a jack or a spade)

4) A basket contains 3 red and 2 yellow apples. Two apples are chosen at random.   
Find the following probabilities:

a) P(one red, one yellow) b) P(at least one red)

5) A basket contains 4 red, 3 white, and 3 blue marbles. Three marbles are chosen at random.   
Find the following probabilities:

a) P(two red, one white) b) P(first red, second white, third blue)

c) P(at least one red) d) P(none red)

6) Given a family of four children. Find the following probabilities:

a) P(All boys) b) P(1 boy and 3 girls)

7) Consider a family of three children. Find the following:

a) P(children of both sexes | first born is a boy) b) P(all girls | children of both sexes)

8) Mrs. Rossetti is flying from San Francisco to New York. On her way to the San Francisco Airport she encounters heavy traffic and determines that there is a 20% chance that she will be late to the airport and will miss her flight. Even if she makes her flight, there is a 10% chance that she will miss her connecting flight at Chicago. What is the probability that she will make it to New York as scheduled?

9) At a college, twenty percent of the students take history, thirty percent take math, and ten percent take both. What percent of the students take at least one of these two courses?

10) In a T-maze, a mouse may run to the right (R) or may run to the left (L). A mouse goes up the maze three times, and events E and F are described as follows:

E: Runs to the right on the first trial F: Runs to the left two consecutive times

Determine whether the events E and F are independent.

11) A college has found that 20% of its students take advanced math courses, 40% take advanced English courses and 15% take both advanced math and advanced English courses. If a student is selected at random, what is the probability that

a) he is taking English given that he is taking math? b) he is taking math or English?

12) If there are 35 students in a class, what is the probability that at least two have the same birthday?

13) A student feels that her probability of passing accounting is .62, of passing mathematics is .45, and her passing accounting or mathematics is .85. Find the probability that the student passes both accounting and math.

***SECTION 8.6 PROBLEM SET: CHAPTER REVIEW***

14) There are nine judges on the U. S. Supreme Court. Suppose that five are conservative and four are liberal. This year the court will act on six major cases. What is the probability that out of six cases the court will favor the conservatives in at least four?

15) Five cards are drawn from a deck. Find the probability of obtaining

a) four cards of a single suit

b) two cards of one suit, two of another suit, and one from the remaining

c) a pair(e.g. two aces and three other cards)

d) a straight flush(five in a row of a single suit but not a royal flush)

16) The following table shows a distribution of drink preferences by gender.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Coke(C) | Pepsi(P) | Seven Up(S) | TOTALS |
| Male(M) | 60 | 50 | 22 | 132 |
| Female(F) | 50 | 40 | 18 | 108 |
| TOTALS | 110 | 90 | 40 | 240 |

The events M, F, C, P and S are defined as Male, Female, Coca Cola, Pepsi, and Seven Up, respectively. Find the following:

a) P(F | S) b) P( P | F)

c) P(C | M) d) P(M | P U C)

e) Are the events F and S mutually exclusive? f) Are the events F and S independent?

17) At a clothing outlet 20% of the clothes are irregular, 10% have at least a button missing and 4% are both irregular and have a button missing. If Martha found a dress that has a button missing, what is the probability that it is irregular?

18) A trade delegation consists of four Americans, three Japanese and two Germans. Three people are chosen at random. Find the following probabilities:

a) P(two Americans and one Japanese) b) P(at least one American)

c) P(One of each nationality) d) P(no German)

19) A coin is tossed three times, and the events E and F are as follows.

E: It shows a head on the first toss F: Never turns up a tail

Are the events E and F independent?

20) If P(E) = .6 and P(F) = .4 and E and F are mutually exclusive, find P(E and F).

21) If P(E)=.5 and P(F)=.3 and E and F are independent, find P(E U F).

22) If P(F)=.9 and P(E | F)=.36 and E and F are independent, find P(E).

23) If P(E)=.4 and P(E or F) =.9 and E and F are independent, find P(F).

24) If P(E) = .4 and P(F | E) = .5, find P(E and F).

25) If P(E) = .6 and P(E and F) = .3, find P(F | E).

26) If P(E ) = .3 and P(F) = .4 and E and F are independent, find P(E | F).