In this chapter, you will learn to:

1. Solve financial problems that involve simple interest.

2. Solve problems involving compound interest.

3. Find the future value of an annuity, and the amount of payments to a sinking fund.

4. Find the future value of an annuity, and an installment payment on a loan.

# 6.1 Simple Interest and Discount

In this section, you will learn to:

1. Find simple interest.

2. Find present value.

3. Find discounts and proceeds.

## *SIMPLE INTEREST*

It costs to borrow money. The rent one pays for the use of money is called the **interest.** The amount of money that is being borrowed or loaned is called the **principal** or **present value**. Simple interest is paid only on the original amount borrowed. When the money is loaned out, the person who borrows the money generally pays a fixed rate of interest on the principal for the time period he keeps the money. Although the interest rate is often specified for a year, it may be specified for a week, a month, or a quarter, etc. The credit card companies often list their charges as monthly rates, sometimes it is as high as 1.5% a month.

|  |
| --- |
| **SIMPLE INTEREST** If an amount P is borrowed for a time t at an interest rate of r per time period, then the simple interest is given by  **I = P. r . t**The total amount A, also called the accumulated value or the future value, is given by A = P + I = P + Prt  or **A = P(1 + rt)**where interest rate r is expressed in decimals. |

 ***Example 1*** Ursula borrows $600 for 5 months at a simple interest rate of 15% per year. Find the interest, and the total amount she is obligated to pay?

 ***Solution:*** The interest is computed by multiplying the principal with the interest rate and the time.

 I = Prt

 I = $600(.15) = $37.50

The total amount is

 A = P + I = $600 + $37.50 = $637.50

Incidentally, the total amount can be computed directly as

 A = P(1 + rt) = $600[1 + (.15)(5/12)]

 = $600(1 + .0625)

 = $637.50

 ***Example 2*** Jose deposited $2500 in an account that pays 6% simple interest. How much money will he have at the end of 3 years?

 ***Solution:*** The total amount or the future value is given by A = P(1 + rt).

 A = $2500[1 + (.06)(3)]

A = $2950

 ***Example 3*** Darnel owes a total of $3060 which includes 12% interest for the three years he borrowed the money. How much did he originally borrow?

 ***Solution:*** This time we are asked to compute the principal P.

 $3060 = P[1 + (.12)(3)]

 $3060 = P(1.36)

 = P

 $2250 = P Darnel originally borrowed $2250.

***Example 4*** A Visa credit card company charges a 1.5% finance charge each month on the unpaid balance. If Martha owed $2350 and has not paid her bill for three months, how much does she owe now?

 ***Solution:*** Before we attempt the problem, the reader should note that in this problem the rate of finance charge is given per month and not per year.

The total amount Martha owes is the previous unpaid balance plus the finance charge.

 A = $2350 + $2350(.015)(3) = $2350 + $105.75 = $2455.75

Alternatively, again, we can compute the amount directly by using formula A = P(1 + rt)

 A = $2350[1 + (.015)(3)] = $2350(1.045) = $2455.75

## DISCOUNTS AND PROCEEDS

Banks often deduct the simple interest from the loan amount at the time that the loan is made. When this happens, we say the loan has been **discounted.** The interest that is deducted is called the **discount**, and the actual amount that is given to the borrower is called the **proceeds**. The amount the borrower is obligated to repay is called the **maturity value.**

|  |
| --- |
| **DISCOUNT AND PROCEEDS** If an amount M is borrowed for a time t at a discount rate of r per year, then the discount D is  **D = M. r . t**The proceeds P, the actual amount the borrower gets, is given by P = M – D P = M – Mrt or **P = M(1 – rt)**where interest rate r is expressed in decimals. |

***Example 5*** Francisco borrows $1200 for 10 months at a simple interest rate of 15% per year. Determine the discount and the proceeds.

 ***Solution:*** The discount D is the interest on the loan that the bank deducts from the loan amount.

 D = Mrt

 D = $1200(.15)() = $150

Therefore, the bank deducts $150 from the maturity value of $1200, and gives Francisco $1050. Francisco is obligated to repay the bank $1200.

In this case, the discount D = $150, and the proceeds P = $1200 – $150 = $1050.

 ***Example 6*** If Francisco wants to receive $1200 for 10 months at a simple interest rate of 15% per year, what amount of loan should he apply for?

 ***Solution:*** In this problem, we are given the proceeds P and are being asked to find the maturity value M.

We have P = $1200, r = .15, t = 10/12 . We need to find M.

We know P = M – D

but also D = Mrt

therefore P = M – Mrt = M(1 – rt)

 $1200 = M[1 – (.15)()]

 We need to solve for M.

 $1200 = M(1 – .125)

 $1200 = M(.875)

 = M

 $1371.43 = M

Therefore, Francisco should ask for a loan for $1371.43.

The bank will discount $171.43 and Francisco will receive $1200.

## SECTION 6.1 SUMMARY

Below is a summary of the formulas we developed for calculations involving simple interest:

**SIMPLE INTEREST**

If an amount P is borrowed for a time t at an interest rate of r per time period, then the simple interest is given by

 **I = P . r . t**

The total amount A also called the accumulated value or the future value is given by

 A = P + I = P + Prt

 or A = P(1 + rt)

where the interest rate r is expressed in decimals.

**DISCOUNT AND PROCEEDS**

If an amount M is borrowed for a time t at a discount rate of r per year, then the discount D is

 D = M **.** r **.** t

The proceeds P, the actual amount the borrower receives at the time the money is borrowed, is given by

 P = M – D

 P = M – Mrt

 or P = M(1 – rt)

where interest rate r is expressed in decimals.

At the end of the loan’s term, the borrower repays the entire maturity amount M.

# 6.2 Compound Interest

In this section you will learn to:

1. Find the future value of a lump-sum.

2. Find the present value of a lump-sum.

3. Find the effective interest rate.

## *COMPOUND INTEREST*

In the last section, we examined problems involving simple interest. Simple interest is generally charged when the lending period is short and often less than a year. When the money is loaned or borrowed for a longer time period, if the interest is paid (or charged) not only on the principal, but also on the past interest, then we say the interest is **compounded.**

Suppose we deposit $200 in an account that pays 8% interest. At the end of one year, we will have $200 + $200(.08) = $200(1 + .08) = $216.

Now suppose we put this amount, $216, in the same account. After another year, we will have $216 + $216(.08) = $216(1 + .08) = $233.28.

So an initial deposit of $200 has accumulated to $233.28 in two years. Further note that had it been simple interest, this amount would have accumulated to only $232. The reason the amount is slightly higher is because the interest ($16) we earned the first year, was put back into the account. And this $16 amount itself earned for one year an interest of $16(.08) = $1.28, thus resulting in the increase. So we have earned interest on the principal as well as on the past interest, and that is why we call it compound interest.

Now suppose we leave this amount, $233.28, in the bank for another year, the final amount will be $233.28 + $233.28(.08) = $233.28(1 + .08) = $251.94.

Now let us look at the mathematical part of this problem so that we can devise an easier way to solve these problems.

After one year, we had $200(1 + .08) = $216

After two years, we had $216(1 + .08)

But $216 = $200(1 + .08), therefore, the above expression becomes

  $200(1+.08)(1+.08) = $200(1+.08)2=$233. 28

After three years, we get

  $233.28(1+.08) = $200(1+.08)(1+.08) (1+.08)

which can be written as

 $200(1 + .08)3 = $251.94

Suppose we are asked to find the total amount at the end of 5 years, we will get

 200(1 + .08)5 = $293.87

We summarize as follows:

|  |  |  |
| --- | --- | --- |
| The original amount | $200 | = $200 |
| The amount after one year | $200(1 + .08)  | = $216 |
| The amount after two years | $200(1 + .08)2 | = $233.28 |
| The amount after three years | $200(1 + .08)3 | = $251.94 |
| The amount after five years | $200(1 + .08)5 | = $293.87 |
| The amount after t years | $200(1 + .08)t |  |

## *COMPOUNDING PERIODS*

Banks often compound interest more than one time a year. Consider a bank that pays 8% interest but compounds it four times a year, or quarterly. This means that every quarter the bank will pay an interest equal to one-fourth of 8%, or 2%.

Now if we deposit $200 in the bank, after one quarter we will have $200(1 + ) or $204.

After two quarters, we will have $200(1 + )2 or $208.08.

After one year, we will have $200(1 + )4 or $216.49.

After three years, we will have $200(1 + )12 or $253.65, etc.

|  |  |  |
| --- | --- | --- |
| The original amount | $200 | = $200 |
| The amount after one quarter | $200(1 + ) | = $204 |
| The amount after two quarters | $200(1 + )2 | = $208.08 |
| The amount after one year | $200(1 + )4 | = $216.49 |
| The amount after two years | $200(1 + )8 | = $234.31 |
| The amount after three years | $200(1 + )12 | = $253.65 |
| The amount after five years | $200(1 + )20 | = $297.19 |
| The amount after t years | $200(1 + )4t |  |

Therefore, if we invest a lump-sum amount of P dollars at an interest rate r, compounded n times a year, then after t years the final amount is given by

 

 The following examples use the compound interest formula 

 ***Example 1*** If $3500 is invested at 9% compounded monthly, what will the future value be in four years?

 ***Solution:*** Clearly an interest of .09/12 is paid every month for four years. The interest is compounded 4×12 = 48 times over the four-year period. We get

 

$3500 invested at 9% compounded monthly will accumulate to $5009.92 in four years.

 ***Example 2*** How much should be invested in an account paying 9% compounded daily for it to accumulate to $5,000 in five years?

 ***Solution:***  We know the future value, but need to find the principal.

 

 $5000 = P (1.568225)

 $3188.32 = P

$3188,32 invested into an account paying 9% compounded daily will accumulate to $5,000 in five years.

***Example 3*** If $4,000 is invested at 4% compounded annually, how long will it take to accumulate to $6,000?

 ***Solution:*** n = 1 because annual compounding means compounding only once per year.
The formula simplifies to  when n = 1.

 

We use logarithms to solve for the value of t because the variable t is in the exponent.

 t = log 1.04 (1.5)

Using the change of base formula we can solve for t:

  years

It takes 10.33 years for $4000 to accumulate to $6000 if invested at 4% interest, compounded annually

 ***Example 4*** If $5,000 is invested now for 6 years what interest rate compounded quarterly is needed to obtain an accumulated value of $8000.

 ***Solution:*** We have n = 4 for quarterly compounding.

 

 We use roots to solve for t because the variable r is in the base, whereas the exponent is a known number.

 

Many calculators have a built in “nth root” key or function. In the TI-84 calculator, this is found in the Math menu. Roots can also be calculated as fractional exponents; if necessary, the previous step can be rewritten as

 

 Evaluating the left side of the equation gives

 

An interest rate of 7.91% is needed in order for $5000 invested now to accumulate to $8000 at the end of 6 years, with interest compounded quarterly.

## *EFFECTIVE INTEREST RATE:*

Banks are required to state their interest rate in terms of an **“effective yield”** ” or **“effective interest rate”**, for comparison purposes. The effective rate is also called the Annual Percentage Yield (APY) or Annual Percentage Rate (APR).

The effective rate is the interest rate compounded annually would be equivalent to the stated rate and compounding periods. The next example shows how to calculate the effective rate.

To examine several investments to see which has the best rate, we find and compare the effective rate for each investment.

Example 5 illustrates how to calculate the effective rate.

***Example 5*** If Bank A pays 7.2% interest compounded monthly, what is the effective interest rate?
If Bank B pays 7.25% interest compounded semiannually, what is the effective interest rate? Which bank pays more interest?

 ***Solution:***  Bank A: Suppose we deposit $1 in this bank and leave it for a year, we will get

 

 r EFF = 1.0744 – 1 = 0.0744

We earned interest of $1.0744 – $1.00 = $.0744 on an investment of $1.

**The effective interest rate is 7.44%,** often referred to as the APY or APR.

Bank B: The effective rate is calculated as
r EFF = 

**The effective interest rate is 7.38%**.

Bank A pays slightly higher interest, with an effective rate of 7.44%, compared to Bank B with effective rate 7.38%.

## *CONTINUOUS COMPOUNDING*

Interest can be compounded yearly, semiannually, quarterly, monthly, and daily.
Using the same calculation methods, we could compound every hour, every minute, and
even every second. As the compounding period gets shorter and shorter, we move toward
 the concept of continuous compounding.

But what do we mean when we say the interest is compounded continuously, and how do we compute such amounts? When interest is compounded "infinitely many times", we say that the interest is **compounded continuously**. Our next objective is to derive a formula to model continuous compounding.

Suppose we put $1 in an account that pays 100% interest. If the interest is compounded once a year, the total amount after one year will be $1(1 + 1) = $2.

If the interest is compounded semiannually, in one year we will have $1(1 + 1/2)2 = $2.25

If the interest is compounded quarterly, in one year we will have $1(1 + 1/4)4 = $2.44

If the interest is compounded monthly, in one year we will have $1(1 + 1/12)12 = $2.61

If the interest is compounded daily, in one year we will have $1(1 + 1/365)365 = $2.71

We show the results as follows:

|  |  |  |
| --- | --- | --- |
| Frequency of compounding | Formula | Total amount |
| Annually | $1(1 + 1) | $2 |
| Semiannually | $1(1 + 1/2)2  | $2.25 |
| Quarterly | $1(1 + 1/4)4  | $2.44140625 |
| Monthly | $1(1 + 1/12)12 | $2.61303529 |
| Daily | $1(1 + 1/365)365  | $2.71456748 |
| Hourly | $1(1 + 1/8760)8760 | $2.71812699 |
| Every minute | $1(1 + 1/525600)525600 | $2.71827922 |
| Every Second | $1(1 + 1/31536000)31536000 | $2.71828247 |
| Continuously | $1(2.718281828...)  | $2.718281828... |

We have noticed that the $1 we invested does not grow without bound. It starts to stabilize to an irrational number 2.718281828... given the name "*e*" after the great mathematician Euler.

In mathematics, we say that as n becomes infinitely large the expression equals *e*.

Therefore, it is natural that the number e play a part in continuous compounding.
It can be shown that as n becomes infinitely large the expression 

Therefore, it follows that if we invest $P at an interest rate r per year, compounded continuously, after t years the final amount will be given by

 A = P. *e*rt .

 ***Example 6*** $3500 is invested at 9% compounded continuously. Find the future value in 4years.

 ***Solution:*** Using the formula for the continuous compounding, we get A = P*e*rt.

 A = $3500*e*.09×4

 A = $3500*e*.36

 A = $5016.65

 ***Example 7*** If an amount is invested at 7% compounded continuously, what is the effective interest rate?

 ***Solution:*** If we deposit $1 in the bank at 7% compounded continuously for one year, and subtract that $1 from the final amount, we get the effective interest rate in decimals.

 r EFF = 1e.07 – 1

 r EFF = 1.0725 – 1

 r EFF =.0725 or 7.25%

 ***Example 8*** If an amount is invested at 7% compounded continuously, how long will it take to double?

We offer two solutions.

Solution 1 uses logarithms to calculate the exact answer, so it is preferred.
 We already used this method in Example 3 to solve for time needed for an investment to accumulate to a specified future value.

Solution 2 provides an estimated solution that is applicable only to doubling time, but not to other multiples. Students should find out from their instructor if there is a preference as to which solution method is to be used for doubling time problems.

 ***Solution: Solution 1:* Calculating the answer exactly:**P*e*.07t = A .

We don’t know the initial value of the prinicipal but we do know that the accumulated value is double (twice) the principal.

 P*e*.07t = 2P

We divide both sides by P

 *e*.07t = 2

Using natural logarithm:

 .07t = ln(2)

 t = ln(2)/.07 = 9.9 years

It takes 9.9 years for money to double if invested at 7% continuous interest.

***Solution 2:* Estimating the answer using the Law of 70:**

The Law of 70 is a useful tool for estimating the time needed for an investment to double in value. It is an approximation and is not exact and comes from our previous solution. We calculated that

 t = ln(2)/ r where r was 0.07 in that solution

Evaluating ln(2) = 0.693, gives t = 0.693/r. Multiplying numerator and denominator by 100 gives t = 69.3/ (100r)

If we estimate 69.3 by 70 and state the interest rate as a percent instead of a decimal, we obtain the Law of 70:

**Law of 70:** The number of years required to double money ≈ 70 ÷ interest rate

* Note that this is an approximate estimate only.
* The interest rate is stated as a percent (not decimal) in the Law of 70.

Using the Law of 70 gives us t ≈ 70/7=10 which is close to but not exactly the value of 9..9 years calculated in Solution 1.

Approximate Doubling Time in Years as a Function of Interest Rate

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Annual interest rate | 1% | 2% | 3% | 4% | 5% | 6% | 7% | 8% | 9% | 10% |
| Number of years to double money | 70 | 35 | 23 | 18 | 14 | 12 | 10 | 9 | 8 | 7 |

The pattern in the table approximates the Law of 70.

 With technology available to do calculations using logarithms, we would use the Law of 70 only for quick estimates of doubling times. Using the Law of 70 as an estimate works only for doubling times, but not other multiples, so it’s not a replacement for knowing how to find exact solutions.

However, the Law of 70 can be useful to help quickly estimate many “doubling time” problems mentally, which can be useful in compound interest applications as well as other applications involving exponential growth.

 ***Example 9*** a. At the peak growth rate in the 1960’s the worlds population had a doubling time of 35 years. At that time, approximately what was the growth rate?

 b. As of 2015, the world population’s annual growth rate was approximately 1.14%. Based on that rate, find the approximate doubling time.

 ***Solution:*** a. According to the law of 70,

 doubling time = 35 ≈ 70 ÷ r

 r ≈ 2 expressed as a percent

 Therefore, the world population was growing at an approximate rate of 2% in the 1960’s.

 b.. According to the law of 70,

 doubling time t ≈ 70 ÷ r = 70 ÷ 1.14 ≈ 61 years

 If the world population were to continue to grow at the annual growth rate of 1.14% , it would take approcimately 61 years for the population to double.

## SECTION 6.2 SUMMARY

Below is a summary of the formulas we developed for calculations involving compound interest:

|  |
| --- |
| **COMPOUND INTEREST n times per year**1. If an amount P is invested for t years at an interest rate r per year, compounded n times a year, then the future value is given by  **P is called the principal and is also called the present value.**2. If a bank pays an interest rate r per year, compounded n times a year, then the effective interest rate is given by **rEFF =** **CONTINUOUSLY COMPOUNDED INTEREST** 3. If an amount P is invested for t years at an interest rate r per year, compounded continuously, then the future value is given by **A = P*e*rt** 4. If a bank pays an interest rate r per year, compounded n times a year, then the effective interest rate is given by **rEFF = *e*r – 1**5. The Law of 70 states that **The number of years to double money is approximately  70 ÷ interest rate** |

# 6.3 Annuities and Sinking Funds

In this section, you will learn to:

1. Find the future value of an annuity.

2. Find the amount of payments to a sinking fund.

## *ORDINARY ANNUITY*

In the first two sections of this chapter, we examined problems where an amount of money was deposited lump sum in an account and was left there for the entire time period. Now we will do problems where timely payments are made in an account. When a sequence of payments of some fixed amount are made in an account at equal intervals of time, we call that an **annuity**. And this is the subject of this section.

To develop a formula to find the value of an annuity, we will need to recall the formula for the sum of a geometric series.

A geometric series is of the form: a + ax + ax2 + ax3+ . . . + axn.

In a geometric series, each subsequent term is obtained by multiplying the preceding term by a number, called the common ratio. A geometric series is completely determined by knowing its first term, the common ratio, and the number of terms.

In the example, a + ax + ax2 + ax 3+ . . . + axn–1 the first term of the series is a, the common ratio is x, and the number of terms is n.

The following are some examples of geometric series.

3 + 6 + 12 + 24 + 48 has first term a = 3 and common ratio x = 2

2 + 6 + 18 + 54 + 162 has first term a = 2 and common ratio x = 3

37 + 3.7 + .37 + .037 + .0037 has first term a = 35 and common ratio x = 0.1

In your algebra class, you developed a formula for finding the sum of a geometric series. You probably used r as the symbol for the ratio, but we are using x because r is the symbol we have been using for the interest rate.

The formula for the sum of a geometric series with first term *a* and common ratio *x* is:

 

We will use this formula to find the value of an annuity.

Consider the following example.

 ***Example 1*** If at the end of each month a deposit of $500 is made in an account that pays 8% compounded monthly, what will the final amount be after five years?

 ***Solution:*** There are 60 deposits made in this account. The first payment stays in the account for 59 months, the second payment for 58 months, the third for 57 months, and so on.

The first payment of $500 will accumulate to an amount of $500(1 + .08/12)59.

The second payment of $500 will accumulate to an amount of $500(1 + .08/12)58.

The third payment will accumulate to $500(1 + .08/12)57.

The fourth payment will accumulate to $500(1 + .08/12)56.

And so on . . .

Finally the next to last (59th) payment will accumulate to $500(1 + .08/12)1.

The last payment is taken out the same time it is made, and will not earn any interest.

To find the total amount in five years, we need to add the accumulated value of these sixty payments.

In other words, we need to find the sum of the following series.

$500(1 + .08/12)59 + $500(1 + .08/12)58 + $500(1 + .08/12)57 + . . . + $500

Written backwards, we have

$500 + $500(1 + .08/12) + $500(1 + .08/12)2 + . . . + $500(1 + .08/12)59

This is a geometric series with a = $500, r = (1 + .08/12), and n = 59. The sum is

 =$500(73.47686)

 = $36738.43

When the payments are made at the end of each period rather than at the beginning, we call it an **ordinary annuity.**

|  |
| --- |
|  **Future Value of an Ordinary Annuity**If a payment of m dollars is made in an account n times a year at an interest r, then the final amount A after t years is  **A =** The future value is also called the accumulated value |

 ***Example 2*** Tanya deposits $300 at the end of each quarter in her savings account. If the account earns 5.75% compounded quarterly, how much money will she have in 4 years?

 ***Solution:*** The future value of this annuity can be found using the above formula.

 A =

 A = $300(17.8463) = $5353.89

If Tanya deposits $300 into a savings account earning 5.75% compounded quarterly for 4 years, then at the end of 4 years she will have $5,353.89

 ***Example 3*** Robert needs $5,000 in three years. How much should he deposit each month in an account that pays 8% compounded monthly in order to achieve his goal?

 ***Solution:*** If Robert saves m dollars per month, after three years he will have

 

But we'd like this amount to be $5,000. Therefore,

 = $5000

 m (40.5356) = $5000
 m =  = $123.35

Robert needs to deposit $123.35 at the end of each month for 3 years into an account paying 8% compounded monthly in order to have $5,000 at the end of 5 years.

## SINKING FUND

When a business deposits money at regular intervals into an account in order to save for a future purchase of equipment, the savings fund is referred to as a “**sinking fund**”. Calculating the sinking fund deposit uses the same method as the previous problem.

 ***Example 4*** A business needs $450,000 in five years. How much should be deposited each quarter in a sinking fund that earns 9% compounded quarterly to have this amount in five years?

 ***Solution:*** Again, suppose that m dollars are deposited each quarter in the sinking fund. After five years, the future value of the fund should be $450,000. This suggests the following relationship:

= $450,000 m (24.9115) = 450,000

 m = = $ 18,063.93

The business needs to deposit $18063.93 at the end of each quarter for 5 years into an sinking fund earning interest of 9% compounded quarterly in order to have $450,000 at the end of 5 years.

## *ANNUITY DUE*

If the payment is made at the beginning of each period, rather than at the end, we call it an **annuity due**. The formula for the annuity due can be derived in a similar manner. Reconsider Example 1, with the change that the deposits are made at the beginning of each month.

 ***Example 5*** If at the beginning of each month a deposit of $500 is made in an account that pays 8% compounded monthly, what will the final amount be after five years?

 ***Solution:*** There are 60 deposits made in this account. The first payment stays in the account for 60 months, the second payment for 59 months, the third for 58 months, and so on.

The first payment of $500 will accumulate to an amount of $500(1 + .08/12)60.

The second payment of $500 will accumulate to an amount of $500(1 + .08/12)59.

The third payment will accumulate to $500(1 + .08/12)58.

And so on . . .

The last payment is in the account for a month and accumulates to $500(1 + .08/12)

To find the total amount in five years, we need to find the sum of the series:

$500(1 + .08/12)60 + $500(1 + .08/12)59 + $500(1 + .08/12)58 + . . . + $500(1 + .08/12)

Written backwards, we have

$500(1 + .08/12) + $500(1 + .08/12)2 + . . . + $500(1 + .08/12)60

If we add $500 to this series, and later subtract that $500, the value will not change. We get

**$500** + $500(1 + .08/12) + $500(1 + .08/12)2 + . . . + $500(1 + .08/12)60 – **$500**

Except for the last term, we have a geometric series with a = $500, r = (1 + .08/12), and n = 60. Therefore the sum is

 A = ­­– $500

 A =$500(74.9667) – $500

 A = $37483.35 – $500

 A = $36983.35

So, in the case of an annuity due, to find the future value, we increase the number of periods n by 1, and subtract one payment.

The Future Value of an **“Annuity Due”**

 A = – m

Most of the problems we are going to do in this chapter involve ordinary annuities, therefore, we will down play the significance of the last formula for the annuity due. We mentioned the formula for the annuity due only for completeness.

## SECTION 6.3 SUMMARY

Finally, it is the author's wish that the student learn the concepts in a way that he or she will not have to memorize every formula. It is for this reason formulas are kept at a minimum.
But before we conclude this section we will once again mention one single equation that will help us find the future value, as well as the sinking fund payment.

|  |
| --- |
| **The Equation to Find the Future Value of an Ordinary Annuity, or the Amount of Periodic Payment to a Sinking Fund**If a payment of m dollars is made in an account n times a year at an interest r, then the future value A after t years is  **A =** The future value is also called the accumulated value.Note that the formula assumes that the payment period is the same as the compounding period. If these are not the same, then this formula does not apply.  |

# 6.4 Present Value of an Annuity and Installment Payment

In this section, you will learn to:

1. Find the present value of an annuity.

2. Find the amount of installment payment on a loan.

## *PRESENT VALUE OF AN ANNUITY*

In Section 6.2, we learned to find the future value of a lump sum, and in Section 6.3, we learned to find the future value of an annuity. With these two concepts in hand, we will now learn to amortize a loan, and to find the present value of an annuity.

The **present value** of an annuity is the amount of money we would need now in order to be able to make the payments in the annuity in the future. In other word, the present value is the value now of a future stream of payments.

We start by breaking this down step by step to understand the concept of the present value of an annuity. After that, the examples provide a more efficient way to do the calculations by working with concepts and calculations we have already explored in Sections 6.2 and 6.3.

Suppose Carlos owns a small business and employs an assistant manager to help him run the business. Assume it is January 1 now. Carlos plans to pay his assistant manager a $1000 bonus at the end of this year and another $1000 bonus at the end of the following year. Carlos’ business had good profits this year so he wants to put the money for his assistant’s future bonuses into a savings account now. The money he puts in now will earn interest at the rate of 4% per year compounded annually while in the savings account.

How much money should Carlos put into the savings account now so that he will be able to withdraw $1000 one year from now and another $1000 two years from now?

At first, this sounds like a sinking fund. But it is different. In a sinking fund, we put money into the fund with periodic payments to save to accumulate to a specified lump sum that is the future value at the end of a specified time period.

In this case we want to put a lump sum into the savings account now, so that lump sum is our principal, P. Then we want to withdraw that amount as a series of period payments; in this case the withdrawals are an annuity with $1000 payments at the end of each of two years.

We need to determine the amount we need in the account now, the present value, to be able to make withdraw the periodic payments later.

We use the compound interest formula from Section 6.2 with r = 0.04 and n = 1 for annual compounding to determine the present value of each payment of $1000.

Consider the first payment of $1000 at the end of year 1. Let P1 be its present value

  so P1=$961.54

Now consider the second payment of $1000 at the end of year 2. Let P2 is its present value

  so P2=$924.56

To make the $1000 payments at the specified times in the future, the amount that Carlos needs to deposit now is the present value 

The calculation above was useful to illustrate the meaning of the present value of an annuity.
But it is not an efficient way to calculate the present value. If we were to have a large number of annuity payments, the step by step calculation would be long and tedious.

Example 1 investigates and develops an efficient way to calculate the present value of an annuity, by relating the future (accumulated) value of an annuity and its present value.

***Example 1*** Suppose you have won a lottery that pays $1,000 per month for the next 20 years. But, you prefer to have the entire amount now. If the interest rate is 8%, how much will you accept?

***Solution:*** This classic present value problem needs our complete attention because the rationalization we use to solve this problem will be used again in the problems to follow.

Consider, for argument purposes, that two people Mr. Cash, and Mr. Credit have won the same lottery of $1,000 per month for the next 20 years. Mr. Credit is happy with his $1,000 monthly payment, but Mr. Cash wants to have the entire amount now.

Our job is to determine how much Mr. Cash should get. We reason as follows:

If Mr. Cash accepts P dollars, then the P dollars deposited at 8% for 20 years should yield the same amount as the $1,000 monthly payments for 20 years.
In other words, we are comparing the future values for both Mr. Cash and Mr. Credit, and we would like the future values to equal.

Since Mr. Cash is receiving a lump sum of x dollars, its future value is given by the lump sum formula we studied in Section 6.2, and it is

 A = P(1 + .08/12)240

Since Mr. Credit is receiving a sequence of payments, or an annuity, of $1,000 per month, its future value is given by the annuity formula we learned in Section 6.3. This value is

 A =

The only way Mr. Cash will agree to the amount he receives is if these two future values are equal. So we set them equal and solve for the unknown.

 P (1 + .08/12)240 =

 P (4.9268) = $1000 (589.02041)

 P (4.9268) = $589020.41

 P = $119, 554.36

The present value of an ordinary annuity of $1,000 each month for 20 years at 8% is $119,554.36

The reader should also note that if Mr. Cash takes his lump sum of P = $119,554.36 and invests it at 8% compounded monthly, he will have an accumulated value of A=$589,020.41 in 20 years.

## *INSTALLMENT PAYMENT ON A LOAN*

If a person or business needs to buy or pay for something now (a car, a home, college tuition, equipment for a business) but does not have the money, they can borrow the money as a loan.

They receive the loan amount called the principal (or present value) now and are obligated to pay back the principal in the future over a stated amount of time (term of the loan), as regular periodic payments with interest.

Example 2 examines how to calculate the loan payment, using reasoning similar to Example 1.

 ***Example 2*** Find the monthly payment for a car costing $15,000 if the loan is amortized over five years at an interest rate of 9%.

 ***Solution:*** Again, consider the following scenario:

Two people, Mr. Cash and Mr. Credit, go to buy the same car that costs $15,000. Mr. Cash pays cash and drives away, but Mr. Credit wants to make monthly payments for five years.

Our job is to determine the amount of the monthly payment. We reason as follows:

If Mr. Credit pays m dollars per month, then the m dollar payment deposited each month at 9% for 5 years should yield the same amount as the $15,000 lump sum deposited for 5 years.

Again, we are comparing the future values for both Mr. Cash and Mr. Credit, and we would like them to be the same.

Since Mr. Cash is paying a lump sum of $15,000, its future value is given by the lump sum formula, and it is

 $15,000(1 + .09/12)60

Mr. Credit wishes to make a sequence of payments, or an annuity, of x dollars per month, and its future value is given by the annuity formula, and this value is

 We set the two future amounts equal and solve for the unknown.

 $15,000(1 + .09/12)60 = 

 $15,000(1.5657) = m(75.4241)

 $311.38 = m

Therefore, the monthly payment needed to repay the loan is $311.38 for five years.

## SECTION 6.4 SUMMARY

We summarize the method used in examples 1 and 2 below.

|  |
| --- |
| **The Equation to Find the Present Value of an Annuity,** **Or the Installment Payment for a Loan**If a payment of m dollars is made in an account n times a year at an interest r, then the present value P of the annuity after t years is  **P(1 + r/n)nt =** When used for a loan, the amount P is the loan amount, and m is the periodic payment needed to repay the loan over a term of t years with n payments per year.If the present value or loan amount is needed, solve for PIf the periodic payment is needed, solve for m.Note that the formula assumes that the payment period is the same as the compounding period. If these are not the same, then this formula does not apply.  |

Finally, we note that many finite mathematics and finance books develop the formula for the present value of an annuity differently.

Instead of using the formula : P(1 + r/n)nt = (Formula 6.4.1)

and solving for the present value P after substituting the numerical values for the other items in the formula, many textbooks first solve the formula for P in order to develop a new formula for the present value. Then the numerical information can be substituted into the present value formula and evaluated, without needing to solve algebraically for P.

**Alternate Method to find Present Value of an Annuity**

 Starting with formula 6.4.1: P(1 + r/n)nt =

 Divide both sides by (1+r/n)nt to isolate P, and simplify

 

**** (Formula 6.4.2)

The authors of this book believe that it is easier to use formula 6.4.1 at the top of this page and solve for P or m as needed. In this approach there are fewer formulas to understand, and many students find it easier to learn. In the problems the rest of this chapter, when a problem requires the calculation of the present value of an annuity, formula 6.4.1 will be used.

However, some people prefer formula 6.4.2, and it is mathematically correct to use that method. Note that if you choose to use formula 6.4.2, you need to be careful with the negative exponents in the formula. And if you needed to find the periodic payment, you would still need to do the algebra to solve for the value of m.

It would be a good idea to check with your instructor to see if he or she has a preference.
 In fact, you can usually tell your instructor’s preference by noting how he or she explains and demonstrates these types of problems in class.

# 6.5 Miscellaneous Application Problems

In this section, you will learn to apply to concepts for compound interest for savings and annuities to:

1. Find the outstanding balance, partway through the term of a loan, of the future payments still remaining on the loan.

2. Perform financial calculations in situations involving several stages of savings and/or annuities.

3. Find the fair market value of a bond.

4. Construct an amortization schedule for a loan.

We have already developed the tools to solve most finance problems.
Now we use these tools to solve some application problems.

## *OUTSTANDING BALANCE ON A LOAN*

One of the most common problems deals with finding the balance owed at a given time during the life of a loan. Suppose a person buys a house and amortizes the loan over 30 years, but decides to sell the house a few years later. At the time of the sale, he is obligated to pay off his lender, therefore, he needs to know the balance he owes. Since most long term loans are paid off prematurely, we are often confronted with this problem.

To find the outstanding balance of a loan at a specified time, we need to find the present value P of all future payments that have not yet been paid. In this case t does not represent the entire term of the loan. Instead:

* t represents the time that still remains on the loan
* nt represents the total number of future payments.

 ***Example 1*** Mr. Jackson bought his house in 1995, and financed the loan for 30 years at an interest rate of 7.8%. His monthly payment was $1260. In 2015, Mr. Jackson decides to pay off the loan. Find the balance of the loan he still owes.

 ***Solution:*** The reader should note that the original amount of the loan is not mentioned in the problem. That is because we don't need to know that to find the balance.

The original loan was for 30 years. 20 years have past so there are $10=30-20$ years still remaining. 12(10) = 120 payments still remain to be paid on this loan.

As for the bank or lender is concerned, Mr. Jackson is obligated to pay $1260 each month for 10 more years; he still owes a total of 120 payments. But since Mr. Jackson wants to pay it all off now, we need to find the present value P at the time of repayment of the remaining 10 years of payments of $1260 each month.
Using the formula we get for the present value of an annuity, we get

 P(1 + .078/12)120 = 

 P (2.17597) = $227957.85

 P = $104761.48

**To Find the Outstanding Balance of a Loan**

If a loan has a payment of m dollars made i n times a year at an interest r,
then the outstanding value of the loan when there are t years still remaining on the loan is given by P:

 **P(1 + r/n)nt =**

IMPORTANT: Note that t is not the original term of the loan but instead
 t is the amount of time still remaining in the future
 nt is the number of payments still remaining in the future

If the problem does not directly state the amount of time still remaining in the term of the loan, then it must be calculated BEFORE using the above formula as
t = original term of loan – time already passed since the start date of the loan.

Note that there are other methods to find the outstanding balance on a loan, but the method illustrated above is the easiest.

One alternate method would be to use an amortization schedule, as illustrated toward the end of this section. An amortization schedule shows the payments, interest, and outstanding balance step by step after each loan payment. An amortization schedule is tedious to calculate by hand but can be easily constructed using spreadsheet software.

Another way to find the outstanding balance, that we will not illustrate here, is to find the difference A – B, where

A = the original loan amount (principal) accumulated to the date on which we want to find the outstanding balance (using compound interest formula)

B = the accumulated value of all payments that have been made as of the date on which we want to find the outstanding balance (using formula for accumulated value of an annuity)

In this case we would need do a compound interest calculation and an annuity calculation; we then need to find the difference between them. Three calculations are needed instead of one.
It is a mathematically acceptable way to calculate the outstanding balance. However,
 **it is very strongly recommended that students use the method explained in box above** and illustrated in Example 1, as it is much simpler.

## *PROBLEMS INVOLVING MULTIPLE STAGES OF SAVINGS AND/OR ANNUITIES*

Consider the following situations:

a. Suppose a baby, Aisha, is born and her grandparents invest $5000 in a college fund.
The money remains invested for 18 years until Aisha enters college, and then is withdrawn in equal semiannual payments over the 4 years that Aisha expects to need to finish college.
The college investment fund earns 5% interest compounded semiannually. How much money can Aisha withdraw from the account every six months while she is in college?

b. Aisha graduates college and starts a job. She saves $1000 each quarter, depositing it into a retirement savings account. Suppose that Aisha saves for 30 years and then retires.
At retirement she wants to withdraw money as an annuity that pays a constant amount every month for 25 years. During the savings phase, the retirement account earns 6% interest compounded quarterly. During the annuity payout phase, the retirement account earns 4.8% interest compounded monthly. Calculate Aisha’s monthly retirement annuity payout.

These problems appear complicated. But each can be broken down into two smaller problems involving compound interest on savings or involving annuities. Often the problem involves a savings period followed by an annuity period. ; the accumulated value from first part of the problem may become a present value in the second part. Read each problem carefully to determine what is needed.

 ***Example 2*** Suppose a baby, Aisha, is born and her grandparents invest $8000 in a college fund. The money remains invested for 18 years until Aisha enters college, and then is withdrawn in equal semiannual payments over the 4 years that Aisha expects to attend college. The college investment fund earns 5% interest compounded semiannually. How much money can Aisha withdraw from the account every six months while she is in college?

 ***Solution:* Part 1: Accumulation of College Savings:**  Find the accumulated value at the end of 18 years of a sum of $8000 invested at 5% compounded semiannually.

 A = $8000(1 + .05/2)(2×18) = $8000(1.025)36 = $8000(2.432535)

A= $19460.28

 **Part 2: Seminannual annuity payout from savings to put toward college expenses.** Find the amount of the semiannual payout for four years using the accumulated savings from part 1 of the problem with an interest rate of 5% compounded semiannually.

A= $19460.28 in Part 1 is the accumulated value at the end of the savings period.
This ecomes the present value P =$19460.28 when calculating the semiannual payments in Part 2.



 $23710.46 = m (8.73612)

m = $2714.07

Aisha will be able to withdraw $2714.07 semiannually for her college expenses.

 ***Example 3***  Aisha graduates college and starts a job. She saves $1000 each quarter, depositing it into a retirement savings account. Suppose that Aisha saves for 30 years and then retires. At retirement she wants to withdraw money as an annuity that pays a constant amount every month for 25 years. During the savings phase, the retirement account earns 6% interest compounded quarterly. During the annuity payout phase, the retirement account earns 4.8% interest compounded monthly. Calculate Aisha’s monthly retirement annuity payout.

 ***Solution:* Part 1: Accumulation of Retirement Savings:**  Find the accumulated value at the end of 30 years of $1000 deposited at the end of each quarter into a retirement savings account earning 6% interest compounded quarterly.



A = $331288.19

 **Part 2: Monthly retirement annuity payout:** Find the amount of the monty annuity payments for 25 years using the accumulated savings from part 1 of the problem with an interest rate of 4.8% compounded monthly.

A= $331288.19 in Part 1 is the accumulated value at the end of the savings period.
This amount will become the present value P =$331288.19 when calculating the monthly retirement annuity payments in Part 2.



$1097285.90 = m (578.04483)

m= $1898.27

Aisha will have a monthly retirement annuity income of $1898.27 when she retires.

## *FAIR MARKET VALUE OF A BOND*

Whenever a business, and for that matter the U. S. government, needs to raise money it does it by selling bonds. A **bond** is a certificate of promise that states the terms of the agreement. Usually the business sells bonds for the **face amount** of $1,000 each for a stated **term**, a period of time ending at a specified **maturity** date.

The person who buys the bond, the **bondholder**, pays $1,000 to buy the bond.

The bondholder is promised two things: First that he will get his $1,000 back at the maturity date, and second that he will receive a fixed amount of interest every six months.

As the market interest rates change, the price of the bond starts to fluctuate. The bonds are bought and sold in the market at their **fair market value**.

The interest rate a bond pays is fixed, but if the market interest rate goes up, the value of the bond drops since the money invested in the bond could earn more if invested elsewhere. When the value of the bond drops, we say it is trading at a **discount**.

On the other hand, if the market interest rate drops, the value of the bond goes up since the bond now yields a higher return than the market interest rate, and we say it is trading at a **premium**.

 ***Example 4*** The Orange Computer Company needs to raise money to expand. It issues a 10-year $1,000 bond that pays $30 every six months. If the current market interest rate is 7%, what is the fair market value of the bond?

 ***Solution:*** The bond certificate promises us two things – An amount of $1,000 to be paid in 10 years, and a semi-annual payment of $30 for ten years. Therefore, to find the fair market value of the bond, we need to find the present value of the lump sum of $1,000 we are to receive in 10 years, as well as, the present value of the $30 semi-annual payments for the 10 years.

We will let P1 = the present value of the (face amount of $1,000

 P1 (1 + .07/2)20 = $1,000

Since the interest is paid twice a year, the interest is compounded twice a year and nt = 2(10)=20

 P1 (1.9898) = $1,000

 P1 = $502.56

We will let P2 = the present value of the $30 semi-annual payments is

 P2 (1 + .07/2)20 = 

 P2 (1.9898) = 848.39

 P2 = $426.37

The present value of the lump-sum $1,000 = $502.56

The present value of the $30 semi-annual payments = $426.37

The fair market value of the bond is P = P1+ P2 = $502.56 + $426.37 = $928.93

Note that because the market interest rate of 7% is higher than the bond’s implied interest rate of 6% implied by the semiannual payments, the bond is selling at a discount; its fair market value of $928.93 is less than its face value of $1000.

 ***Example 5*** A state issues a 15 year $1000 bond that pays $25 every six months. If the current market interest rate is 4%, what is the fair market value of the bond?

 ***Solution:*** The bond certificate promises two things – an amount of $1,000 to be paid in 15 years, and semi-annual payments of $25 for 15 years. To find the fair market value of the bond, we find the present value of the $1,000 face value we are to receive in 15 years and add it to the present value of the $25 semi-annual payments for the 15 years. In this example, nt = 2(15)=30.

We will let P1 = the present value of the lump-sum $1,000

 P1(1 + .04/2)30 = $1,000

 P1 = $552.07

We will let P2 = the present value of the $25 semi-annual payments is

 P2 (1 + .04/2)30 = 

 P2 (1.18114) = $1014.20

 P2 = $559.90

The present value of the lump-sum $1,000 = $552.07

The present value of the $30 semi-annual payments = $559.90

Therefore, the fair market value of the bond is
P = P1+ P2 =$552.07 + $559.90 = $1111.97

Because the market interest rate of 4% is lower than the interest rate of 5% implied by the semiannual payments, the bond is selling at a premium: the fair market value of $1,111.97 is more than the face value of $1,000.

To summarize:

**To Find the Fair Market Value of a Bond:**

Find the present value of the face amount A that is payable at the maturity date:

**A = P1 (1 + r/n)nt** ; solve to find P1

Find the present value of the semiannually payments of $m over the term of the bond:

 **P2(1 + r/n)nt =** ; solve to find P2

The fair market value (or present value or price or current value) of the bond is the sum of the present values calculated above:

 P = **P1+ P2**

## *AMORTIZATION SCHEDULE FOR A LOAN*

An amortization schedule is a table that lists all payments on a loan, splits them into the portion devoted to interest and the portion that is applied to repay principal, and calculates the outstanding balance on the loan after each payment is made.

***Example 6*** An amount of $500 is borrowed for 6 months at a rate of 12%. Make an amortization schedule showing the monthly payment, the monthly interest on the outstanding balance, the portion of the payment contributing toward reducing the debt, and the outstanding balance.

 ***Solution:*** The reader can verify that the monthly payment is $86.27.

The first month, the outstanding balance is $500, and therefore, the monthly interest on the outstanding balance is

 (outstanding balance)(the monthly interest rate) = ($500)(.12/12) = $5

This means, the first month, out of the $86.27 payment, $5 goes toward the interest and the remaining $81.27 toward the balance leaving a new balance of $500 – $81.27 = $418.73.

Similarly, the second month, the outstanding balance is $418.73, and the monthly interest on the outstanding balance is ($418.73)(.12/12) = $4.19. Again, out of the $86.27 payment, $4.19 goes toward the interest and the remaining $82.08 toward the balance leaving a new balance of $418.73 – $82.08 = $336.65. The process continues in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Payment # | Payment | Interest | Debt Payment |  Balance |
| 1 | $86.27 | $5 | $81.27 | $418.73 |
| 2 | $86.27 | $4.19 | $82.08 | $336.65 |
| 3 | $86.27 | $3.37 | $82.90 | $253.75 |
| 4 | $86.27 | $2.54 | $83.73 | $170.02 |
| 5 | $86.27 | $1.70 | $84.57 | $85.45 |
| 6 | $86.27 | $0.85 | $85.42 | $0.03 |

Note that the last balance of 3 cents is due to error in rounding off.

An amortization schedule is usually lengthy and tedious to calculate by hand. For example, an amortization schedule for a 30 year mortgage loan with monthly payments would have (12)(30)=360 rows of calculations in the amortization schedule table. A car loan with 5 years of monthly payments would have 12(5)=60 rows of calculations in the amortization schedule table. However it would be straightforward to use a spreadsheet application on a computer to do these repetitive calculations by inputting and copying formulas for the calculations into the cells.

Most of the other applications in this section's problem set are reasonably
straightforward, and can be solved by taking a little extra care in interpreting them. And remember, there is often more than one way to solve a problem.

# 6.6 Classification of Finance Problems

In this section, you will review the concepts of chapter 6 to:

1. re-examine the types of financial problems and classify them.

2. re-examine the vocabulary words used in describing financial calculations

We'd like to remind the reader that the hardest part of solving a finance problem is determining the category it falls into. So in this section, we will emphasize the classification of problems rather than finding the actual solution.

We suggest that the student read each problem carefully and look for the word or words that may give clues to the kind of problem that is presented. For instance, students often fail to distinguish a lump-sum problem from an annuity. Since the payments are made each period, an annuity problem contains words such as each, every, per etc.. One should also be aware that in the case of a lump-sum, only a single deposit is made, while in an annuity numerous deposits are made at equal spaced time intervals. To help interpret the vocabulary used in the problems, we include a glossary at the end of this section.

Students often confuse the present value with the future value. For example, if a car costs $15,000, then this is its present value. Surely, you cannot convince the dealer to accept $15,000 in some future time, say, in five years. Recall how we found the installment payment for that car. We assumed that two people, Mr. Cash and Mr. Credit, were buying two identical cars both costing $15, 000 each. To settle the argument that both people should pay exactly the same amount, we put Mr. Cash's cash of $15,000 in the bank as a lump-sum and Mr. Credit's monthly payments of x dollars each as an annuity. Then we make sure that the future values of these two accounts are equal. As you remember, at an interest rate of 9%

 the future value of Mr. Cash's lump-sum was $15,000(1 + .09/12)60, and

 the future value of Mr. Credit's annuity was .

To solve the problem, we set the two expressions equal and solve for m.

The present value of an annuity is found in exactly the same way. For example, suppose Mr. Credit is told that he can buy a particular car for $311.38 a month for five years, and Mr. Cash wants to know how much he needs to pay. We are finding the present value of the annuity of $311.38 per month, which is the same as finding the price of the car. This time our unknown quantity is the price of the car. Now suppose the price of the car is P, then

 the future value of Mr. Cash's lump-sum is P(1 + .09/12)60, and

 the future value of Mr. Credit's annuity is .

Setting them equal we get,

 P(1 + .09/12)60 =

 P(1.5657) = ($311.38) (75.4241)

 P(1.5657) = $23,485.57

 P = $15,000.04

## CLASSIFICATION OF PROBLEMS AND EQUATIONS FOR SOLUTIONS

We now list six problems that form a basis for all finance problems.
Further, we classify these problems and give an equation for the solution.

 ***Problem 1*** If $2,000 is invested at 7% compounded quarterly, what will the final amount be in 5 years?

 ***Classification:* Future (accumulated) Value of a Lump-sum** or FV of a lump-sum.

 ***Equation:*** FV = A = $2000(1 + .07/4)20.

 ***Problem 2*** How much should be invested at 8% compounded yearly, for the final amount to be $5,000 in five years?

 ***Classification:* Present Value of a Lump-sum** or PV of a lump-sum.

 ***Equation:*** PV(1 + .08)5 = $5,000

 ***Problem 3*** If $200 is invested *each* month at 8.5% compounded monthly, what will the final amount be in 4 years?

 ***Classification:* Future (accumulated) Value of an Annuity** or FV of an annuity.

 ***Equation:*** FV = A =

 ***Problem 4*** How much should be invested *each* month at 9% for it to accumulate to $8,000 in three years?

 ***Classification:* Sinking Fund Payment**

 ***Equation:*** = $8,000

 ***Problem 5*** Keith has won a lottery paying him $2,000 *per* month for the next 10 years. He'd rather have the entire sum now. If the interest rate is 7.6%, how much should he receive?

 ***Classification:* Present Value of an Annuity** or PV of an annuity.

 ***Equation:*** PV(1 + .076/12)120 =

 ***Problem 6*** Mr. A has just donated $25,000 to his alma mater. Mr. B would like to donate an equivalent amount, but would like to pay by monthly payments over a five year period. If the interest rate is 8.2%, determine the size of the monthly payment?

 ***Classification:* Installment Payment**.

 ***Equation:*** = $25,000(1 + .082/12)60.

## GLOSSARY: VOCABULARY AND SYMBOLS USED IN FINANCIALCALCULATIONS

As we’ve seen in these examples, it’s important to read the problems carefully to correctly identify the situation. It is essential to understand to vocabulary for financial problems. Many of the vocabulary words used are listed in the glossary below for easy reference.

|  |  |  |
| --- | --- | --- |
| t | Term | Time period for a loan or investment. In this book t is represented in years and should be converted into years when it is stated in months or other units. |
| P | Principal | Principal is the amount of money borrowed in a loan.If a sum of money is invested for a period of time, the sum invested at the start is the Principal.  |
| P | Present Value | Value of money at the beginning of the time period. |
| A | Accumulated ValueFuture Value | Value of money at the end of the time period |
| D | Discount | In loans involving simple interest, a discount occurs if the interest is deducted from the loan amount at the beginning of the loan period, rather than being repaid at the end of the loan period.  |
| m | Periodic Payment | The amount of a constant periodic payment that occurs at regular intervals during the time period under consideration (examples: periodic payments made to repay a loan, regular periodic payments into a bank account as savings, regular periodic payment to a retired person as an annuity,) |
| n | Number of payment periods and compounding periods per year | In this book, when we consider periodic payments, we will always have the compounding period be the same as the payment period. In general the compounding and payment periods do not have to be the same, but the calculations are more complicated if they are different. If the periods differ, formulas for the calculations can be found in finance textbooks or various online resources. Calculations can easily be done using technology such as an online financial calculator, or financial functions in a spreadsheet, or a financial pocket calculator.  |
| nt | Number of periods | nt = (number of periods per year)×(number of years) nt gives the total number of payment and compounding periods In some situations we will calculate nt as the multiplication shown above. In other situations the problem may state nt, such as a problem describing an investment of 18 months duration compounded monthly. In this example: nt = 18 months and n = 12; then t = 1.5 years but t is not stated explicitly in the problem. The TI-84+ calculators built in TVM solver uses N = nt. |
| r | Annual interest rateNominal rate | The stated annual interest rate. This is stated as a percent but converted to decimal form when using financial calculation formulas. If a bank account pays 3% interest compounded quarterly, then 3% is the nominal rate, and it is included in the financial formulas as r = 0.03 |
| r/n | Interest rate per compounding period | If a bank account pays 3% interest compounded quarterly, then r/n = 0.03/4 = 0. 075, corresponding to a rate of 0.75% per quarter. Some Finite Math books use the symbol i to represent r/n |

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| rEFF | Effective RateEffective Annual Interest RateAPY Annual Percentage YieldAPR Annual Percentage Rate | The effective rate is the interest rate compounded annually that would give the same interest rate as the compounded rate stated for the investment. The effective rate provides a uniform way for investors or borrowers to compare different interest rates with different compounding periods. |
| I  | Interest | Money paid by a borrower for the use of money borrowed as a loan.Money earned over time when depositing money into a savings account, certificate of deposit, or money market account. When a person deposits money in a bank account, the person depositing the funds is essentially temporarily lending the money to the bank and the bank pays interest to the depositor.  |
|  | Sinking Fund | A fund set up by making payments over a period of time into a savings or investment account in order to save to fund a future purchase. Businesses use sinking funds to save for a future purchase of equipment at the end of the savings period by making periodic installment payments into a sinking fund. |
|  | Annuity | An annuity is a stream of periodic payments. In this book it refers to a stream of constant periodic payments made at the end of each compounding period for a specific amount of time.In common use the term annuity generally refers to a constant stream of periodic payments received by a person as retirement income, such as from a pension.Annuity payments in general may be made at the end of each payment period (ordinary annuity) or at the start of each period (annuity due).The compounding periods and payment periods do not need to be equal, but in this textbook we only consider situations when these periods are equal. |
|  | Lump Sum | A single sum of money paid or deposited at one time, rather than being spread out over time. An example is lottery winnings if the recipient chooses to receive a single “lump sum” one-time payment, instead of periodic payments over a period of time or as. Use of the word lump sum indicates that this is a one time transaction and is not a stream of periodic payments.  |
|  | Loan | An amount of money that is borrowed with the understanding that the borrower needs to repay the loan to the lender in the future by the end of a period of time that is called the term of the loan. The repayment is most often accomplished through periodic payments until the loan has been completely repaid over the term of the loan.However there are also loans that can be repaid as a single sum at the end of the term of the loan, with interest paid either periodically over the term or in a lump sum at the end of the loan or as a discount at the start of the loan.  |