

**PROBLEMS INVOLVING MULTIPLE STAGES OF SAVINGS AND/OR ANNUITIES**

Consider the following situations:

a. Suppose a baby, Aisha, is born and her grandparents invest \$8000 in a college fund. The money remains invested for 18 years until Aisha enters college, and then is withdrawn in equal semiannual payments over the 4 years that Aisha expects to need to finish college. The college investment fund earns 5% interest compounded semiannually. How much money can Aisha withdraw from the account every six months while she is in college?

b. Aisha graduates college and starts a job. She saves \$1000 each quarter, depositing it into a retirement savings account. Suppose that Aisha saves for 30 years and then retires. At retirement she wants to withdraw money as an annuity that pays a constant amount every month for 25 years. During the savings phase, the retirement account earns 6% interest compounded quarterly. During the annuity payout phase, the retirement account earns 4.8% interest compounded monthly. Calculate Aisha’s monthly retirement annuity payout.

These problems appear complicated. But each can be broken down into two smaller problems involving compound interest on savings or involving annuities. Often the problem involves a savings period followed by an annuity period. ; the accumulated value from first part of the problem may become a present value in the second part. Read each problem carefully to determine what is needed.

◆ **Example 2** Suppose a baby, Aisha, is born and her grandparents invest \$8000 in a college fund. The money remains invested for 18 years until Aisha enters college; then it is withdrawn in equal semiannual payments over the 4 years that Aisha expects to attend college. The college investment fund earns 5% interest compounded semiannually. How much can Aisha withdraw from the account every six months while she is in college?

**Solution:** **Part 1: Accumulation of College Savings:** Find the accumulated value at the end of 18 years of a sum of \$8000 invested at 5% compounded semiannually.

$$A = \$8000(1 + .05/2)^{(2 \times 18)} = \$8000(1.025)^{36} = \$8000(2.432535)$$

$$A = \$19460.28$$

**Part 2: Semiannual annuity payout from savings to put toward college expenses.** Find the amount of the semiannual payout for four years using the accumulated savings from part 1 of the problem with an interest rate of 5% compounded semiannually.

A = \$19460.28 in Part 1 is the accumulated value at the end of the savings period. This becomes the present value P = \$19460.28 when calculating the semiannual payments in Part 2.

$$\$19460.28 \left(1 + \frac{.05}{2}\right)^{2 \times 4} = \frac{m \left[ \left(1 + \frac{.05}{2}\right)^{2 \times 4} - 1 \right]}{(.05 / 2)}$$

$$\$23710.46 = m (8.73612)$$

$$m = \$2714.07$$

Aisha will be able to withdraw \$2714.07 semiannually for her college expenses.