Chapter 3 Section 1
Systems of Linear Equations in Two Variables

Systems of Linear Equations and Their Solutions
Two linear equations are called a system of linear equations or a linear system.
A solution of a system of linear equations is an ordered pair that satisfied both equations in the system.

Example:
Ordered pair (3, 4) satisfies the system

$$\begin{cases} x+y=7 & (3+4 \textit{ is } 7) \\ x-y=-1 & (3-4 \textit{ is } -1) \end{cases}$$

Writing the solution
Solution: x = 3 and y = 4
Set notation: {(3, 4)}
Ordered pair: (3, 4)

Example 1: page 179:
Determine if each ordered pair is a solution to the system
a) (-3, -2) b) (1, -4)

$$\begin{cases} x+2y=-7 \\ 2x-3y=0 \end{cases}$$

When two lines are graphed, there are three situations.
What are the situations?

The Number of Solutions to a System of Two Linear Equations

The number of solutions to a system of two linear equations in two variables is given by one of the following. (See **Figure 3.3**.)

Number of Solutions	What This Means Graphically
Exactly one ordered-pair solution	The two lines intersect at one point.
No solution	The two lines are parallel.
Infinitely many solutions	The two lines are identical.

A system of linear equations can have exactly one solution, no solution, or infinitely many solutions.

Vocabulary:
Inconsistent system – no solution – parallel lines
Dependent system – infinitely many solutions – lines coincide
Consistent system – at least one solution.

Solving Linear Systems by Graphing

Solution is the coordinates of the point of intersection, (x, y)

Graph both equations on the same set of axes and determine to coordinates of the point of intersection.

1) $\begin{cases} y = -x - 1 \\ 4x - 3y = 24 \end{cases}$

2) $\begin{cases} y = -2x + 4 \\ 7x - 2y = 3 \end{cases}$

Problems with this method?

Elimination Method – Substitution

$\begin{cases} y = -2x + 4 \\ 7x - 2y = 3 \end{cases}$

1) Solve either of the equations for one variable in terms of the other.
Since the first equation is solved for y, the next step is followed.

2) Substitute the expression from step 1 into the other equation.
Replace y in the second equation with the expression in step 1

$7x - 2(\mathbf{-2x + 4}) = 3$ since $y = \mathbf{-2x + 4}$

3) Solve the equation for the variable.
$7x - 2(\mathbf{-2x + 4}) = 3$
$x = 1$

4) Replace the value found in step 3 into the first equation to solve for the other value.
$y = -2x + 4$
$y = -2(\mathbf{1}) + 4$
$y = 2$

The intersection is (1, 2)

5) Check the solution in both of the systems given equations.

Try:

$$\begin{cases} 3x+2y=4 \\ 2x+y=1 \end{cases}$$

The substitution method is useful when one can isolate a variable easily, but what if one cannot isolate a variable?

Elimination Method – Addition Method

$$\begin{cases} 3x-4y=11 \\ -3x+2y=-7 \end{cases}$$

Looking that this system, one can eliminate a variable by adding the two equations. Thus, the x variable is eliminated.

$$\begin{cases} 3x-4y=11 \\ -3x+2y=-7 \end{cases}$$

Add -2y = 4

y = -2

To find the other coordinate, one would substitute y = -2 into one of the original equations. Thus x = 1 and the intersection is (1, -2).

The key step is to obtain one of the variables coefficients that are opposites. One might need to multiply one or both equations by some nonzero number so that the coefficients of one of the variables, x or y, become opposites.

Example 5: page 183

$$\begin{cases} 3x+4y=-10 \\ 5x-2y=18 \end{cases}$$

We must rewrite one or both equations in equivalent form to obtain the coefficients of the same variable, the opposites of each other.

Looking at the equations, one can obtain the coefficients of y, the opposites by multiplying the second equation by 2.

$$\begin{cases} 3x+4y=-10 \\ 5x-2y=18 \quad \text{x 2} \end{cases} \longrightarrow \begin{cases} 3x+4y=-10 \\ 10x-4y=36 \end{cases}$$

Since the coefficients of y's are opposites, one can add the equations.

13x = 26

x = 2

Try: $\begin{cases} 4x-7y=-16 \\ 2x+5y=9 \end{cases}$

$$\begin{cases} \dfrac{x}{2}-5y=32 \\ \dfrac{3x}{2}-7y=45 \end{cases}$$

No Solution

$$\begin{cases} 3x-2y=6 \\ 6x-4y=18 \end{cases}$$

0 = 6

Many Solutions

$$\begin{cases} y=3x-2 \\ 15x-5y=10 \end{cases}$$

10 = 10