

Chapter 2 Section 3 The Algebra of Functions

Domain of a Function: largest set of real numbers for which the value of $f(x)$ is a real number.

Example 1: page 127

Find the domain of the function.

a) $f(x) = 3x + 2$

b) $g(x) = \frac{3x+2}{x+1}$

What values of x makes the function defined?

Domain of f : $(-\infty, \infty)$

b) Since it is a fraction, what values of x makes the function not defined?

When $x = -1$, the fraction is not defined, so one has to eliminate this value from the domain.

Domain of g : $(-\infty, -1) \cup (-1, \infty)$, union, or

Find the domain: $f(x) = \frac{1}{x+5}$

Algebra of Functions

Combine function using addition, subtraction, multiplication, and division.

Sum: $f + g$: $(f + g)(x) = f(x) + g(x)$

Difference: $f - g$: $(f - g)(x) = f(x) - g(x)$

Product: fg : $(fg)(x) = f(x) \cdot g(x)$

Quotient: $\frac{f}{g}$: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

Example: $f(x) = 2x$, $g(x) = x - 1$

$$\begin{aligned} (f+g)(x) &= f(x) + g(x) \\ &= 2x + (x - 1) \\ &= 3x - 1 \end{aligned}$$

$$\begin{aligned} (f-g)(x) &= f(x) - g(x) \\ &= 2x - (x - 1) \\ &= 2x - x + 1 \\ &= x + 1 \end{aligned}$$

$$\begin{aligned} (fg)(x) &= f(x) \cdot g(x) \\ &= 2x \cdot (x - 1) \\ &= 2x^2 - 2x \end{aligned}$$

$$\begin{aligned} \left(\frac{f}{g}\right)(x) &= \frac{f(x)}{g(x)} \\ &= \frac{2x}{x-1}, x \neq 1 \end{aligned}$$

Example 2: page 128

c) $f(x) = x^2 - 3$, $g(x) = 4x + 5$, find $(f + g)(3)$