## SCALAR (DOT) PRODUCT

Before we define work in physics we need to first define the scalar (dot) product between two vectors. The reason for this is because we will define work in terms of the scalar (dot) product between the force vector and displacement vector.
$\mathbf{A} \bullet \mathbf{B}=A B \cos \theta$ (Scalar Product)
$\mathbf{A} \bullet \mathbf{B}=\mathbf{B} \cdot \mathbf{A}$ (Commutative Law)
$\mathbf{A} \cdot \mathbf{( B}+\mathbf{C})=\mathbf{A} \cdot \mathbf{B}+\mathbf{A} \cdot \mathbf{C} \quad$ (Distributive Law)


A

Note that the scalar product is a scalar quantity and not a vector quantity!
Ex. $1 \mathbf{A} \perp \mathbf{B}$


Ex. A parallel to B


Ex. A antiparallel to B


## Dot Product Between Unit Vectors

| $\hat{i} \bullet \hat{i}=\|\hat{i}\|\|\hat{i}\| \cos 0=1$ | $\hat{i} \bullet \hat{j}=0$ |
| :--- | :--- |
| $\hat{j} \bullet \hat{j}=1$ | $\hat{i} \bullet \hat{k}=0$ |
| $\hat{k} \bullet \hat{k}=1$ | $\hat{j} \bullet \hat{k}=0$ |

Given two vectors $\mathbf{A}$ and $\mathbf{B}$ :
$\mathbf{A}=A_{x} \hat{i}+A_{y} \hat{j}+A_{z} \hat{k}$
$\mathbf{B}=B_{x} \hat{i}+B_{y} \hat{j}+B_{z} \hat{k}$
Then:
$\mathbf{A} \bullet \mathbf{B}=A_{x} B_{x}+A_{y} B_{y}+A_{z} B_{z} \quad$ (Scalar Product Between Two Vectors)
Ex.
$\mathbf{A} \bullet \mathbf{A}=A_{x}^{2}+A_{y}^{2}+A_{z}^{2}=A^{2}$
$A=\sqrt{\mathbf{A} \bullet \mathbf{A}} \quad$ (Magnitude of Vector $\mathbf{A}$ )

## WORK DONE BY A CONSTANT FORCE

Def: The work done on an object by a constant force $\mathbf{F}$ is equal to the product of the component of force in the direction of displacement and the magnitude of the displacement of the point of application of the force.


$$
\begin{aligned}
& w=(F \cos \theta) s=F s \cos \theta \\
& w=\mathbf{F} \bullet \mathbf{s} \text { Work Done By A Constant Force }
\end{aligned}
$$

*You can think of work as a measure of the effectiveness of a force to cause an object to be displaced.

Note that work is a scalar quantity and not a vector quantity. Work does not have any direction in space.

## Units of Work

| System | F | S | W |
| :--- | :--- | :--- | :--- |
| SI | N | m | 1 Joule(J)=1 N.m |
| cgs | dyne | cm | 1 erg = 1 dyne.cm |
| British Eng. | lb | ft | 1ft.lb = 1 foot.pound |

