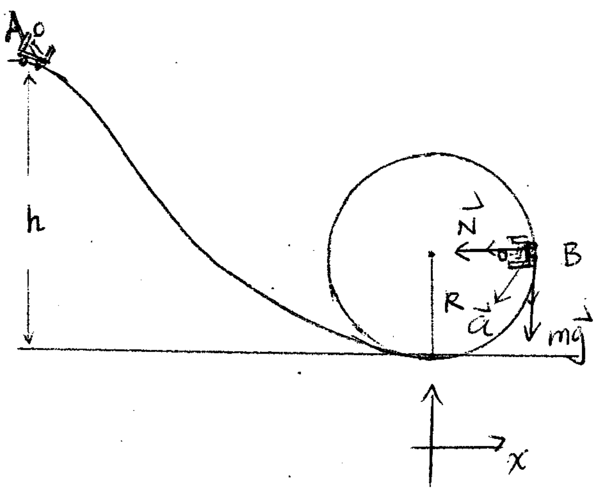
Physics 4A Exam II Name: Solutions

Fall' 09 Lab: 61 62 63

1. (25 points) A roller coaster car of mass m is released from rest at point A at height h and moves freely with negligible friction into a circular loop of radius R in a vertical plane. If h = 3R, when the car is at point B on the circular track, find (a) the normal force on the car by the track, (b) the total acceleration of the car.



from A to B

mechanical energy is conserved,

$$E_A = E_B$$

$$mgh = mgR + \frac{1}{2}mv^2$$

$$mg3R - mgR = \frac{1}{2}mv^2$$

$$v = 2\sqrt{gR}$$

a). $N = ma_c = m\frac{v^2}{R}$

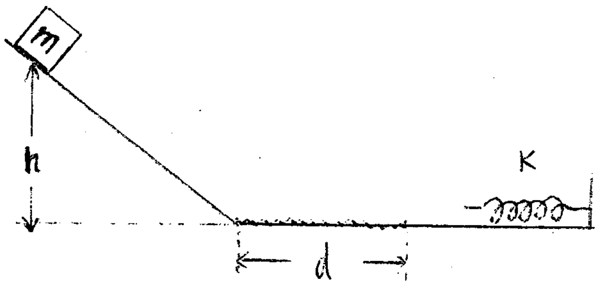
$N = m\frac{4gR}{R} = 4mg$ to the center

b). $a_c = \frac{v^2}{R} = 4g$

$a_t = g$

total $\vec{a} = -4g\hat{i} - g\hat{j}$

2. (25 points) A block of mass m is released from rest at height h above the horizontal surface. The track is frictionless, except for the rough portion of distance d, the coefficient of kinetic friction between the block and the rough surface is μk. The block then hits a spring of constant k, and compresses the spring an amount x from its equilibrium position before coming to rest momentarily. Find x.



friction does work

$$W_{nc} = E_f - E_i$$

$$W_f = \frac{1}{2} k x^2 - mgh$$

spring is compressed x

$$W_f = \vec{f_K} \cdot \Delta \vec{x} = f_K \cdot d \cos 180 = - f_K d = -\mu_K N d$$

$$W_f = -\mu_K m g d$$

$$-\mu_K m g \cdot d = \frac{1}{2} k x^2 - mgh$$

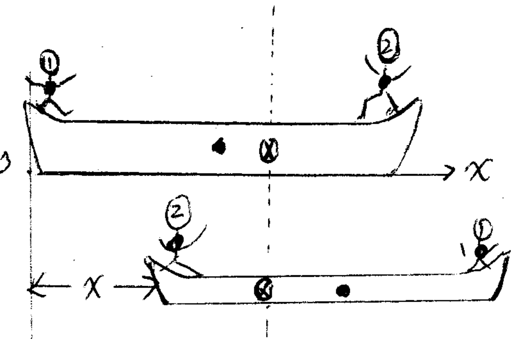
$$\frac{1}{2} k x^2 = mgh - \mu_K m g d$$

$$x = \sqrt{\frac{2mg(h - \mu_K d)}{k}}$$

3. (25 points) A boat of mass M and length L. Child 1 of mass m_1 and child 2 of mass m_2 ,stand on the left end and right end of the boat respectively, the system remains stationary, then they walk to the opposite end of the boat. If there is no friction between water and the boat. Find how far does the boat move during this process and in which direction when (a) $m_1 < m_2$, (b) $m_1 = m_2$.

system $m_1 + m_2 + M$

$$\cancel{\vec{F}_{net\,ext}} \to 0 = (m_1 + m_2 + M)\,\vec{a}_{cm}$$



$$a_{cm} = 0.$$

$$V_{cm} = C = 0$$

$$X_{cm} = C \quad (\text{remains stationary})$$

before: $$X_{cm} = \frac{m_1 X_1 + m_2 X_2 + m_3 X_3}{m_1 + m_2 + m_3} = \frac{m_1 \cdot 0 + m_2 L + M\frac{L}{2}}{m_1 + m_2 + M}$$

after: $$X_{cm} = \frac{m_1 (L + x) + m_2 x + M(\frac{L}{2} + x)}{m_1 + m_2 + M}$$

$$m_2 L + M\frac{L}{2} = m_1 (L + x) + m_2 x + M(\frac{L}{2} + x)$$

$$m_2 L + \cancel{M\frac{L}{2}} = m_1 L + m_1 x + m_2 x + \cancel{M\frac{L}{2}} + Mx$$

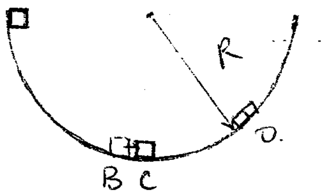
$$(m_2 - m_1) L = (m_1 + m_2 + M)\,x$$

$$x = \frac{m_2 - m_1}{m_1 + m_2 + M} L$$

(a) boat moves to the right if $m_1 < m_2$

(b) if $m_1 = m_2$. $x = 0$. boat doesn't move.

4. (25 points) Two identical masses, one is released from rest in a smooth hemispherical bowl of radius R, the second one rests on the bottom of the bowl. No friction between the masses and the surface of the bowl. If they stick together when they collide, how high above the bottom of the bowl will the masses go after colliding?



from A to B.

$$E_A = E_B$$

$$mgR = \frac{1}{2}mV_B^2 \qquad V_B = \sqrt{2gR}$$

perfectly inelastic collision at C

$$mV_B + m \cdot 0 = (m+m)V_C$$

$$mV_B = 2mV_C$$

$$V_C = \frac{1}{2}V_B = \frac{1}{2}\sqrt{2gR}$$

after collision. two masses stick together

$$E_C = E_D$$

$$\frac{1}{2}(\cancel{2m})V_C^2 = (\cancel{2m}) \cdot g \cdot h$$

$$h = \frac{V_C^2}{2g} = \frac{\frac{1}{4}\cancel{2g}R}{\cancel{2g}} = \frac{1}{4}R$$

Extra credit: (5 points)

describe a car accident in a word collision

write a formula to show the interaction force acted on the car

$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{or} \quad \vec{F}_{ave} \cdot \Delta t = \Delta \vec{p} = m(\vec{V}_f - \vec{V}_i)$$

and explain how an airbag works in a sentence

increases interaction time and decreases force on the passengers