Math 11Exam 3 WINTER 2019 NAME Last__________ First KEY

1. [*5 points*] How much money should be invested now as a lump sum in order to accumulate to $100,000 at the end of 10 years. The interest rate is 3.6% compounded quarterly.

Show work

$A = 100000 \quad t = 10 \quad r = .036 \quad n = 4 \quad$ FIND P

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$

$$100000 = P\left(1+\frac{.036}{4}\right)^{4*10}$$

$$100000 = P(1.43102)$$

$$\boxed{P = \$69880.23}$$

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2 [*6 points*] **Compare effective rates to determine which investment has a higher return.**

Investment A: 7.2% compounded continuously

Show work

$$r_{EFF} = e^{.072} - 1 = .0747$$

Answer: Effective rate is 7 . 4 7 %

Investment B: 7.3% compounded semiannually

Show work

$$r_{EFF} = \left(1+\frac{.073}{2}\right)^2 - 1 = .0743$$

Answer: Effective rate is 7 . 4 3 %.

Answer: Which investment is better? A

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3. [*6 points*] Kareem has a student loan with a 10 year term, at 6.2% interest, compounded monthly. His monthly loan payment is $190.

After making 4 years of payments, he has saved enough money to pay off the outstanding balance of the loan.

Find the outstanding loan balance after he has made payments for 4 years. *Show work*

Given: $r = .062 \quad n = 12 \quad m = 190$

Find P when $t - k = 10 - 4 = 6$ years still remain on the loan

$$P\left(1+\frac{r}{n}\right)^{n(t-k)} = m\frac{\left[\left(1+\frac{r}{n}\right)^{n(t-k)} - 1\right]}{(r/n)}$$

$$P\left(1+\frac{.062}{12}\right)^{12*6} = 190\frac{\left[\left(1+\frac{.062}{12}\right)^{12*6} - 1\right]}{(.062/12)}$$

$$P(1.49924) = 16520.60 \longrightarrow \boxed{P = \$11399.49}$$

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4. [*5 points*] Mr. Huang has an auto repair business. They will need to replace some equipment at the end of 7 years. The cost of the new equipment when it is replaced in 7 years will be $50,000.

The business deposits money into a **sinking fund** at the end of each quarter at 4.4% interest compounded quarterly, for the next 7 years. **Find the quarterly payment** into the sinking fund needed for 7 years to accumulate to $50,000.

Show work.

$t = 7 \quad n = 4 \quad A = 50000 \quad r = .044 \quad$ Find m

$$A = m\frac{\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{(r/n)}$$

$$50000 = m\frac{\left[\left(1+\frac{.044}{4}\right)^{4*7} - 1\right]}{(.044/4)}$$

$$50000 = m(32.58315)$$

$$\boxed{m = \$1534.54}$$

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5. [*7 points*] Ms. Lee wants to buy a car that costs $28,000.
She has $10,000 for a down payment and gets a loan for the rest of the cost of the car.
The loan has a 5 year term with an interest rate of 5.4% compounded monthly.
Find the monthly loan payment. *Show work*

Price = 28000 DownPayment = 10,000

P = Loan = Price - DownPayment = 28000 - 10000 = 18000

r = .054 n = 12 t = 5 Find m

$$P\left(1+\frac{r}{n}\right)^{nt} = m\frac{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{(r/n)}$$

$$18000\left(1+\frac{.054}{12}\right)^{12*5} = m\frac{\left(\left(1+\frac{.054}{12}\right)^{12*5}-1\right)}{(.054/12)}$$

$$23565.08281 = m(68.70473) \quad \boxed{m = \$342.99}$$

6. [*8 points*] A bond with a 10 year term pays $18 semiannually.
When the bond matures at the end of 10 years, the face value of $1,000 is payable.
The market interest rate is 4.2% compounded semiannually.
Find the current fair market value of the bond. (present value, current price) *Show work.*

t = 10 n = 2 m = 18 A = 1000 r = .042

Semiannual Payments

$$P_1\left(1+\frac{r}{n}\right)^{nt} = m\frac{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{(r/n)}$$

$$P_1\left(1+\frac{.042}{2}\right)^{2*10} = 18\frac{\left[\left(1+\frac{.042}{2}\right)^{2*10}-1\right]}{(.042/2)}$$

$$P_1(1.51536) = 441.73422$$

$$P_1 = 291.50$$

FaceValue At Maturity

$$A = P_2\left(1+\frac{r}{n}\right)^{nt}$$

$$1000 = P_2\left(1+\frac{.042}{2}\right)^{2*10}$$

$$1000 = P_2(1.51536)$$

$$P_2 = 659.91$$

$$P = P_1 + P_2 = 291.50 + 659.91$$

$$\boxed{P = \$951.41}$$

7. [*8 points*] Aisha is investing a lump sum deposit of $25,000 for 12 years.
Find the accumulated value at the end of 12 years for the following interest rates:

a. 4.2% interest compounded continuously *Show work*

$$A = Pe^{rt} = 25000e^{.042*12} = \$41383.23$$

b. 4.2% simple interest *Show work*

$$A = P(1+rt) = 25000(1+.042*12) = \$37600.00$$

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8. [*8 points*] Ms. Shah is saving up to retire 20 years from now. For 20 years, at the end of each quarter, she deposits $2900 into a retirement account earning interest of 5.6%, compounded quarterly.

When Ms. Shah retires, she wants to withdraw money from her retirement account at the end of each month for 22 years. The interest rate for the retirement annuity is 4.8% per year compounded monthly.

Find the amount that Ms. Shah can withdraw at the end of each month for 22 years during retirement. *Show work.*

Savings Period

$$A = m\frac{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{(r/n)} = 2900\frac{\left(\left(1+\frac{.056}{4}\right)^{4\times 20}-1\right)}{(.056/4)}$$

$A = 422807.55$ (will become P in next step)

Withdrawal Annuity Period

$$P\left(1+\frac{r}{n}\right)^{nt} = m\frac{\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{(r/n)}$$ Find m

$$422807.55\left(1+\frac{.048}{12}\right)^{12\times 22} = m\frac{\left[\left(1+\frac{.048}{12}\right)^{12\times 22}-1\right]}{(.048/12)}$$

$1212950.046 = m(467.19985)$

$\boxed{m = \$2596.21}$

9 [*7 points*] Antler National Park has 400 deer living in the park.. 10 years later, the deer population has increased to 552 deer. Assume the deer population follows and exponential growth or decay model.

a. **Find the annual percentage growth rate** for the deer population in Antler National Park.
Use function form $y = ab^t$
Answer as a percent, to 2 decimal places hundredths of a percent (which is 4 decimal places in decimal form). You must show correct algebraic work solving for the growth rate.

$552 = 400\, b^{10}$

$\frac{552}{400} = b^{10}$

$1.38 = b^{10}$

$b = \sqrt[10]{1.38} = 1.38^{(1/10)} = 1.38^{.1} = 1.0327$

ANSWER: Growth rate is: 3 . 2 7 %

b. **Write the exponential growth function** for y = population as a function of time t.

Answer: Growth function is : y = $400(1.0327^t)$

10. [*4 points*] An exponential function is $y = a(1.17)^t$. The initial value is 850.

Write the function in the form $y = ae^{kt}$, using the value of k accurate to 3 decimal places.

$b = e^k$

$k = \ln b$ $\quad k = \ln(1.17) = .157$

ANSWER: Growth function in the requested form is : y = $850e^{.157t}$

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11. A restaurant buys new kitchen equipment for \$80,000.
The equipment depreciates with exponential decay at the rate of 17% per year. *Use function form* $y = ab^t$

a. [*3 points*] **Write the exponential decay function** that gives the value of this equipment y as a function of time t in years.

$r = -.17$

$b = 1 + r = 1 + (-.17) = .83$

ANSWER: Decay function is : $y = 80000(.83^t)$

b. [*3 points*] Find the value of the equipment after 3 years.

$y = 80000(.83^3) = \$45742.96$

c. [*4 points*] **Find the amount of time until the equipment is worth only \$30,000.**
State answer to hundredths of a year (2 decimal places). You must show correct algebraic work solving for time.

$30000 = 80000(.83^t)$

$\frac{30000}{80000} = .83^t$

$.375 = .83^t$

$t = \log_{.83}(.375) = \frac{\ln(.375)}{\ln(.83)} = 5.26$ years

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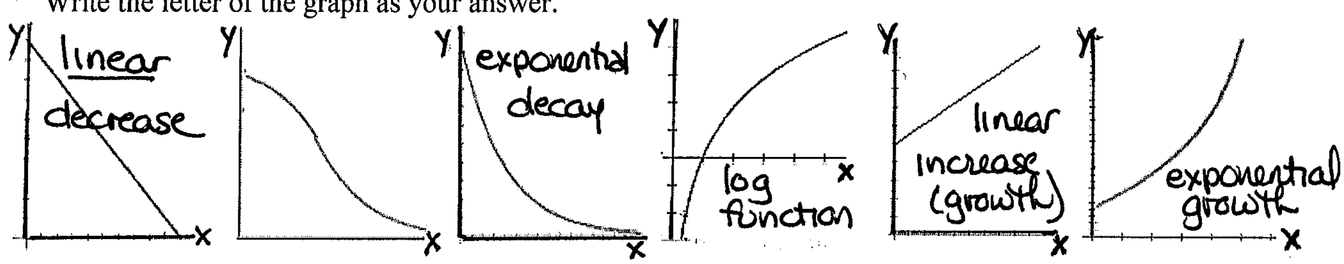
All parts of Question 12 refer to the following graphs.

12. For each part of this question, select the best graph from the choices given below.
Write the letter of the graph as your answer.



Graph A Graph B Graph C Graph D Graph E Graph F

(Part 1). [*3 points*] The value of a fishing boat is modeled by the function $y = 75000(0.93^x)$
The best graph of this situation is Graph C $b = .93 < 1$ so exponential decay

(Part 2) [*3 points*] The value of a painting by a famous artist is modeled by the function $y = 100000e^{0.12x}$
The best graph of this situation is Graph F e^{kt} $k = .12 > 0$ so exponential growth

(Part 3) [*3 points*] $y = 10 \log x$. The best graph of this situation is Graph D