## SECTION 7.1 PROBLEM SET: SETS AND COUNTING

Find the indicated sets.

|  |  |
| --- | --- |
| 1) List all subsets of the following set.  { Al, Bob} | 2) List all subsets of the following set.  { Al, Bob, Chris} |
| 3) List the elements of the following set.  { Al, Bob, Chris, Dave}  {Bob, Chris, Dave, Ed} | 4) List the elements of the following set.  { Al, Bob, Chris, Dave}  {Bob, Chris, Dave, Ed} |

Problems 5 – 8: Let Universal set U = {a, b, c, d, e, f, g, h, i, j}, sets V = {a, e, i, f, h}, W = { a, c, e, g, i}.

List the members of the following sets.

|  |  |
| --- | --- |
| 5) V  W | 6) V  W |
| 7) | 8) |

Problems 9 – 12: Let Universal set U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}  
 and sets A = {1, 2, 3, 4, 5}, B = { 1, 3, 4, 6}, and C = {2, 4, 6}.

List the members of the following sets.

|  |  |
| --- | --- |
| 9) A  B | 10) A  C |
| 11) | 12) |

***SECTION 7.1 PROBLEM SET: SETS AND COUNTING***

Use Venn Diagrams to find the number of elements in the following sets.

|  |  |
| --- | --- |
| 13) In Mrs. Yamamoto's class of 35 students,  12 students are taking history, 18 are taking English, and 4 are taking both. Draw a Venn diagram and use it determine how many students are taking neither history nor English. | 14) In a survey of 1200 college students, 700 used Spotify to listen to music and 400 used iTunes to listen to music; of these, 100 used both.  a. Draw a Venn Diagram and find the number of people in each region of the diagram.  b. How many used either Spotify or iTunes? |
| 15) A survey of athletes revealed that for their minor aches and pains, 30 used aspirin,  50 used ibuprofen, and 15 used both.  How many athletes were surveyed? | 16) In 2016, 80 college students were surveyed about what video services they subscribed to. Suppose the survey showed that 50 use Amazon Prime, 30 use Netflix, 20 use Hulu. Of those, 13 use Amazon Prime and Netflix, 9 use Amazon Prime and Hulu, 7 use Netflix and Hulu. 3 students use all three services.  a. Draw a Venn Diagram and use it to determine the number of people in each region of the diagram.  b. How many use at least one of these?  c. How many use none of these? |

***SECTION 7.1 PROBLEM SET: SETS AND COUNTING***

|  |  |
| --- | --- |
| 17) A survey of 100 students at a college finds that 50 take math, 40 take English, and 30 take history. Of these 15 take English and math,  10 take English and history, 10 take math and history, and 5 take all three subjects. Draw a Venn diagram and find the numbers in each region. Use the diagram to answer the questions below.  a) Find the number of students taking math  but not the other two subjects.  b) The number of students taking English or math but not history.  c) The number of students taking none of these subjects. | 18) In a survey of investors it was found that 100 invested in stocks, 60 in mutual funds, and 50 in bonds. Of these, 35 invested in stocks and mutual funds, 30 in mutual funds and bonds,  28 in stocks and bonds, and 20 in all three. Draw a Venn diagram and find the numbers in each region. Use the diagram to answer the questions below.    a) Find the number of investors that participated in the survey.  b) How many invested in stocks or mutual funds but not in bonds?  c) How many invested in exactly one type of investment? |

***SECTION 7.1 PROBLEM SET: SETS AND COUNTING***

19) A survey of 100 students at a college finds that 50 take math, 40 take English, and 30 take history. Of these 15 take English and math, 10 take English and history, 10 take math and history, and 5 take all three subjects. (This question relates back to question #17.)  
For each of the following draw a Venn Diagram and shade the indicated sets and determine the number of students in the set.

|  |  |
| --- | --- |
| a. Students who take at least one of these classes | b. Students who take exactly one of these classes |
| c. Students who take at least two of these classes | d. Students who take exactly two of these classes |
| e. Students who take at most two of these classes | f. Students who take English or Math but not both |
| g. Students who take Math or History but not English | h. Students who take all of these classes |

***SECTION 7.1 PROBLEM SET: SETS AND COUNTING***

20) In a survey of investors it was found that 100 invested in stocks, 60 in mutual funds, and 50 in bonds. Of these, 35 invested in stocks and mutual funds, 30 in mutual funds and bonds, 28 in stocks and bonds, and 20 in all three. (This question relates back to question #18.)   
For each of the following draw a Venn Diagram and shade the indicated sets and determine the number of students in the set.

|  |  |
| --- | --- |
| a. Investors who invested in mutual funds only | b. Investors who invested in stocks and bonds but not mutual funds |
| c. Investors who invested in exactly one of these investments | d. Investors who invested in exactly two of these investments |
| e. Investors who invested in at least two of these investments | f. Investors who invested in at most two of these investments |
| g. Investors who did not invest in bonds | h. Investors who invested in all three investments. |

## SECTION 7.2 PROBLEM SET: TREE DIAGRAMS AND THE MULTIPLICATION AXIOM

Do the following problems using a tree diagram or the multiplcation axiom.

|  |  |
| --- | --- |
| 1) A man has 3 shirts, and 2 pairs of pants.  Use a tree diagram to determine the number of possible outfits. | 2) In a city election, there are 2 candidates for mayor, and 3 for supervisor.  Use a tree diagram to find the number of ways to fill the two offices. |
| 3) There are 4 roads from Town A to Town B, 2 roads from Town B to Town C.  Use a tree diagram to find the number of ways one can travel from Town A to Town C. | 4) Brown Home Construction offers a selection  of 3 floor plans, 2 roof types, and 2 exterior wall types. Use a tree diagram to determine  the number of possible homes available. |
| 5) For lunch, a small restaurant offers 2 types of soups, three kinds of sandwiches, and two types of soft drinks. Use a tree diagram to determine the number of possible meals consisting of a soup, sandwich, and a soft drink. | 6) A California license plate consists of a number from 1 to 5, then three letters followed by three digits. How many such plates are possible? |

***SECTION 7.2 PROBLEM SET: TREE DIAGRAMS AND THE MULTIPLICATION AXIOM***

Do the following problems using the Multiplication Axiom.

|  |  |
| --- | --- |
| 7) A license plate consists of three letters followed by three digits. How many license plates are possible if no letter may be repeated? | 8) How many different 4-letter radio station call letters can be made if the first letter must be  K or W and no letters can be repeated? |
| 9) How many seven-digit telephone numbers are possible if the first two digits cannot be ones  or zeros? | 10) How many 3-letter word sequences can be formed using the letters {a, b, c, d} if no letter is to be repeated? |

Use a tree diagram for questions 11 and 12:

|  |  |
| --- | --- |
| 11) A family has two children, use a tree diagram to determine all four possibilities of outcomes by gender. | 12) A coin is tossed three times and the sequence  of heads and tails is recorded. Use a tree diagram to list all the possible outcomes. |

***SECTION 7.2 PROBLEM SET: TREE DIAGRAMS AND THE MULTIPLICATION AXIOM***

Do the following problems using the Multiplication Axiom.

|  |  |
| --- | --- |
| 13) In how many ways can a 4-question true-false test be answered? | 14) In how many ways can three people be arranged to stand in a straight line? |
| 15) A combination lock is opened by first turning to the left, then to the right, and then to the left again. If there are 30 digits on the dial, how many possible combinations are there? | 16) How many different answers are possible for a multiple-choice test with 10 questions and five possible answers for each question? |
| 17) In the past, a college required students to use a 4 digit PIN (Personal Identification Number) as their password for its registration system.  How many different PINs are possible if each must have 4 digits with no restrictions on selection or arrangement of the digits used? | 18) The college decided that a more secure password system is needed. New passwords must have 3 numerical digits followed by 6 letters. There are no restrictions on the selection of the numerical digits. However, the letters I and O are not permitted. How many different passwords are possible? |

## SECTION 7.3 PROBLEM SET: PERMUTATIONS

Do the following problems using permutations.

|  |  |
| --- | --- |
| 1) How many three-letter words can be made using the letters {a, b, c, d, e} if no repetitions are allowed? | 2) A grocery store has five checkout counters, and seven clerks. How many different ways can  the 7 clerks be assigned to the 5 counters? |
| 3) A group of fifteen people who are members of an investment club wish to choose a president, and a secretary. How many different ways can this be done? | 4) Compute the following.  a) 9P2  b) 6P4  c) 8P3  d) 7P4 |
| 5) In how many ways can the letters of the word CUPERTINO be arranged if each letter is used only once in each arrangement? | 6) How many permutations of the letters of the word PROBLEM end in a vowel? |
| 7) How many permutations of the letters of the word SECURITY end in a consonant? | 8) How many permutations of the letters PRODUCT have consonants in the second and third positions? |

***SECTION 7.3 PROBLEM SET: PERMUTATIONS***

Do the following problems using permutations.

|  |  |
| --- | --- |
| 9) How many three-digit numbers are there? | 10) How many three-digit odd numbers are there? |
| 11) In how many different ways can five people be seated in a row if two of them insist on sitting next to each other? | 12) In how many different ways can five people be seated in a row if two of them insist on not sitting next to each other? |
| 13) In how many ways can 3 English, 3 history,  and 2 math books be set on a shelf, if the English books are set on the left, history books in the middle, and math books on the right? | 14) In how many ways can 3 English, 3 history,  and 2 math books be set on a shelf, if they are grouped by subject? |
| 15) You have 5 math books and 6 history books to put on a shelf with five slots. In how many ways can you put the books on the shelf if the first two slots are to be filled with math books and the next three with history books? | 16) You have 5 math books and 6 history books to put on a shelf with five slots. In how many ways can you put the books on the shelf if the first two slots are to be filled with the books of one subject and the next three slots are to be filled with the books of the other subject? |

***SECTION 7.3 PROBLEM SET: PERMUTATIONS***

Do the following problems using permutations.

|  |  |
| --- | --- |
| 17) A bakery has 9 different fancy cakes. In how many ways can 5 of the 9 fancy cakes be lined up in a row in the bakery display case? | 18) A landscaper has 6 different flowering plants. She needs to plant 4 of them in a row in a garden. How many different ways can 4 of the 6 plants be arranged in a row? |
| 19) At an auction of used construction vehicles, there are 7 different vehicles for sale. In how many orders could these 7 vehicles be listed in the auction program? | 20) A landscaper has 6 different flowering plants and 4 different non-flowering bushes. She needs to plant a row of 6 plants in a garden. There must be a bush at each end, and four flowering plants in a row in between the bushes. How many different arrangements in a row are possible? |
| 21) In how many ways can all 7 letters of the word QUIETLY be arranged if the letters  Q and U must be next to each other in the order QU? | 22) a. In how many ways can the letters ABCDEXY be arranged if the X and Y  must be next to each other in either order XY or YX?  b. In how many ways can the letters ABCDEXY be arranged if the X and Y can not be next to each other? |

## SECTION 7.4 PROBLEM SET: CIRCULAR PERMUTATIONS AND PERMUTATIONS WITH SIMILAR ELEMENTS

Do the following problems using the techniques learned in this section.

|  |  |
| --- | --- |
| 1) In how many different ways can five children hold hands to play "Ring Around the Rosy"? | 2) In how many ways can three people be made to sit at a round table? |
| 3) In how many different ways can six children ride a "Merry Go Around" with six horses? | 4) In how many ways can three couples be seated at a round table, so that men and women sit alternately? |
| 5) In how many ways can six trinkets be arranged on a chain? | 6) In how many ways can five keys be put on a key ring? |
| 7) Find the number of different permutations of the letters of the word MASSACHUSETTS. | 8) Find the number of different permutations of the letters of the word MATHEMATICS. |

***SECTION 7.4 PROBLEM SET: CIRCULAR PERMUTATIONS AND   
PERMUTATIONS WITH SIMILAR ELEMENTS***

|  |  |
| --- | --- |
| 9) Seven flags are to be flown on seven poles: 3 flags are red, 2 are white, and 2 are blue,. How many different arrangements are possible? | 10) How many different ways can 3 pennies, 2 nickels and 5 dimes be arranged in a row? |
| 11) How many four-digit numbers can be made using two 2's and two 3's? | 12) How many five-digit numbers can be made using two 6's and three 7's? |
| 13) If a coin is tossed 5 times, how many different outcomes of 3 heads and 2 tails are possible? | 14) If a coin is tossed 10 times, how many different outcomes of 7 heads and 3 tails are possible? |
| 15) If a team plays ten games, how many different outcomes of 6 wins and 4 losses are possible? | 16) If a team plays ten games, how many different ways can the team have a winning season? |

## SECTION 7.5 PROBLEM SET: COMBINATIONS

Do the following problems using combinations.

|  |  |
| --- | --- |
| 1) How many different 3-people committees can be chosen from ten people? | 2) How many different 5-player teams can be chosen from eight players? |
| 3) In how many ways can a person choose to vote for three out of five candidates on a ballot for a school board election? | 4) Compute the following:  a) 9C2    b) 6C4    c) 8C3    d) 7C4 |
| 5) How many 5-card hands can be chosen from a deck of cards? | 6) How many 13-card bridge hands can be chosen from a deck of cards? |
| 7) There are twelve people at a party. If they all shake hands, how many different hand-shakes are there? | 8) In how many ways can a student choose to do four questions out of five on a test? |

***SECTION 7.5 PROBLEM SET: COMBINATIONS***

Do the following problems using combinations.

|  |  |
| --- | --- |
| 9) Five points lie on a circle. How many chords can be drawn through them? | 10) How many diagonals does a hexagon have? |
| 11) There are five teams in a league. How many games are played if every team plays each other twice? | 12) A team plays 15 games a season. In how many ways can it have 8 wins and 7 losses? |
| 13) In how many different ways can a 4-child family have 2 boys and 2 girls? | 14) A coin is tossed five times. In how many ways can it fall three heads and two tails? |
| 15) The shopping area of a town is a square that is six blocks by six blocks. How many different routes can a taxi driver take to go from one corner of the shopping area to the opposite cater-corner? | 16) If the shopping area in the previous problem has a rectangular form of 5 blocks by 3 blocks, then how many different routes can a taxi driver take to drive from one end of the shopping area to the opposite kitty corner end? |

***SECTION 7.5 PROBLEM SET: COMBINATIONS***

Do the following problems using combinations.

|  |  |
| --- | --- |
| 17) A team of 7 workers is assigned to a project.  In how many ways can 3 of the 7 workers be selected to make a presentation to the management about their progress on the project? | 18) A real estate company has 12 houses listed for sale by their clients. In how many ways can 5 of the 12 houses be selected to be featured in an advertising brochures? |
| 19) A frozen yogurt store has 9 toppings to choose from. In how many ways can 3 of the 9 toppings be selected ? | 20) A kindergarten teacher has 14 books about a holiday. In how many ways can she select 4 of the books to read to her class in the week before the holiday? |

## SECTION 7.6 PROBLEM SET: COMBINATIONS INVOLVING SEVERAL SETS

Following problems involve combinations from several different sets.

|  |  |
| --- | --- |
| 1) How many 5-people committees consisting of three boys and two girls can be chosen from a group of four boys and four girls? | 2) A club has 4 men, 5 women, 8 boys and 10 girls as members. In how many ways can a group of 2 men, 3 women, 4 boys and 4 girls be chosen? |
| 3) How many 4-people committees chosen from  4 men and 6 women will have at least 3 men? | 4) A batch contains 10 transistors of which three are defective. If three are chosen, in how many ways can they be selected with two defective? |
| 5) In how many ways can five counters labeled A, B, C, D and E at a store be staffed by two men and three women chosen from a group of four men and six women? | 6) How many 4-letter word sequences consisting of two vowels and two consonants can be made from the letters of the word PHOENIX if no letter is repeated? |

Three marbles are chosen from an urn that contains 5 red, 4 white, and 3 blue marbles.   
How many samples of the following type are possible?

|  |  |
| --- | --- |
| 7) All three white. | 8) Two blue and one white. |
| 9) One of each color. | 10) All three of the same color. |
| 11) At least two red. | 12) None red. |

***SECTION 7.6 PROBLEM SET: COMBINATIONS INVOLVING SEVERAL SETS***

Following problems involve combinations from several different sets.

Five coins are chosen from a bag that contains 4 dimes, 5 nickels, and 6 pennies. How many samples of five coins of the following types are possible?

|  |  |
| --- | --- |
| 13) At least four nickels. | 14) No pennies. |
| 15) Five of a kind. | 16) Four of a kind. |
| 17) Two of one kind and two of another kind. | 18) Three of one kind and two of another kind. |

Find the number of different ways to draw a 5-card hand from a deck to have the following combinations.

|  |  |
| --- | --- |
| 19) Three face cards. | 20) A heart flush (all hearts). |
| 21) Two hearts and three diamonds. | 22) Two cards of one suit, and three of another suit. |
| 23) Two kings and three queens. | 24) 2 cards of one value and 3 of another value. |

***SECTION 7.6 PROBLEM SET: COMBINATIONS INVOLVING SEVERAL SETS***

The party affiliation of the 100 United States Senators in the 114th Congress, January 2015, was:   
 44 Democrats, 54 Republicans, and 2 Independents.

|  |  |
| --- | --- |
| 25) In how many ways could a 10 person committee be selected if it is to contain 4 Democrats, 5 Republicans, and 1 Independent? | 26) In how many different ways could a 10 person committee be selected with 6 or 7 Republicans and the rest Democrats (with no Independents)? |

The 100 United States Senators in the 114th Congress, January 2015, included 80 men and 20 women. Suppose a committee senators is working on legislation about wage discrimination by gender.

|  |  |
| --- | --- |
| 27) In how many ways could a 12 person committee be selected to contain equal numbers  of men and women. | 28) In how many ways could a 6 person committee be selected to contain fewer women than men? |

Jorge has 6 rock songs, 7 rap songs and 4 country songs that he likes to listen to while he exercises.   
He randomly selects six (6) of these songs to create a playlist to listen to today while he exercises.

How many different playlists of 6 songs can be selected that satisfy each of the following:  
 (We care which songs are selected to be on the playlist, but not what order they are selected or listed in.)

|  |  |
| --- | --- |
| 29) Playlist has 2 songs of each type | 30) Playlist has no country songs |
| 31) Playlist has 3 rock, 2 rap, and 1 country song | 32) Playlist has 3 or 4 rock songs and all the rest  are rap songs |

## SECTION 7.7 PROBLEM SET: BINOMIAL THEOREM

Use the Binomial Theorem to do the following problems.

|  |  |
| --- | --- |
| 1) Expand (a + b)5. | 2) Expand (a – b)6. |
| 3) Expand (x - 2y)5. | 4) Expand (2x - 3y)4. |
| 5) Find the third term of (2x - 3y)6. | 6) Find the sixth term of (5x + y)8. |

***SECTION 7.7 PROBLEM SET: BINOMIAL THEOREM***

Use the Binomial Theorem to do the following problems.

|  |  |
| --- | --- |
| 7) Find the coefficient of the x3y4 term in the expansion of (2x + y)7. | 8) Find the coefficient of the a4b6 term in the expansion of (3a - b)10. |
| 9) A coin is tossed 5 times, in how many ways is it possible to get three heads and two tails? | 10) A coin is tossed 10 times, in how many ways is it possible to get seven heads and three tails? |
| 11) How many subsets are there of a set that has 6 elements? | 12) How many subsets are there of a set that has n elements? |

## SECTION 7.8 PROBLEM SET: CHAPTER REVIEW

1) Suppose of the 4,000 freshmen at a college everyone is enrolled in a mathematics or an English class during a given quarter. If 2,000 are enrolled in a mathematics class, and 3,000 in an English class, how many are enrolled in both a mathematics class and an English class?

2) In a survey of 250 people, it was found that 125 had read Time magazine, 175 had read Newsweek, 100 had read U. S. News, 75 had read Time and Newsweek, 60 had read Newsweek and U. S. News, 55 had read Time and U. S. News, and 25 had read all three.

a) How many had read Time but not the other two?

b) How many had read Time or Newsweek but not the U. S. News And World Report?

c) How many had read none of these three magazines?

3) At a manufacturing plant, a product goes through assembly, testing, and packing. If a plant has three assembly stations, two testing stations, and two packing stations, in how many different ways can a product achieve its completion?

4) Six people are to line up for a photograph. How many different lineups are possible if three of them insist on standing next to each other ?

5) How many four-letter word sequences can be made from the letters of the word CUPERTINO?

6) In how many different ways can a 20-question multiple choice test be designed so that its answers contain 2 A's, 4 B's, 9 C's, 3 D's, and 2 E's?

7) The U. S. Supreme Court has nine judges. In how many different ways can the judges cast a six-to-three decision in favor of a ruling?

8) In how many different ways can a coach choose a linebacker, a guard, and a tackle from five players on the bench, if all five can play any of the three positions?

9) How many three digit even numbers can be formed from the digits 1, 2, 3, 4, 5 if no repetitions are allowed?

10) Compute: a) 9C4 b) 8P3 c)

11) In how many ways can 3 English, 3 Math, and 4 Spanish books be set on a shelf if the books are grouped by subject?

12) In how many ways can a 10-question multiple choice test with four possible answers for each question be answered?

13) On a soccer team three fullbacks can play any of the three fullback positions, left, center, and right. The three halfbacks can play any of the three halfback positions, the four forwards can play any of the four positions, and the goalkeeper plays only his position. How many different arrangements of the 11 players are possible?

14) From a group of 6 people, 3 are assigned to cleaning, 2 to hauling and one to garbage collecting. How many different ways can this be done?

15) How many three-letter word sequences can be made from the letters of the word OXYGEN?

***SECTION 7.8 PROBLEM SET: CHAPTER REVIEW***

16) In how many ways can 3 books be selected from 4 English and 2 History books if at least one English book must be chosen?

17) Five points lie on the rim of a circle. Choosing the points as vertices, how many different triangles can be drawn?

18) A club consists of six men and nine women. In how many ways can a president, a vice president and a treasurer be chosen if the two of the officers must be women?

19) Of its 12 sales people, a company wants to assign 4 to its Western territory, 5 to its Northern territory, and 3 to its Southern territory. How many ways can this be done?

20) How many permutations of the letters of the word OUTSIDE have consonants in the first and last place?

21) How many distinguishable permutations are there in the word COMMUNICATION?

22) How many five-card poker hands consisting of the following distribution are there?

a) A flush(all five cards of a single suit)

b) Three of a kind(e.g. three aces and two other cards)

c) Two pairs(e.g. two aces, two kings and one other card)

d) A straight(all five cards in a sequence)

23) Company stocks on an exchange are given symbols consisting of three letters. How many different three-letter symbols are possible?

24) How many four-digit odd numbers are there?

25) In how many ways can 7 people be made to stand in a straight line? In a circle?

26) A United Nations delegation consists of 6 Americans, 5 Russians, and 4 Chinese. Answer the following questions.

a) How many committees of five people are there?

b) How many committees of three people consisting of at least one American are there?

c) How many committees of four people having no Russians are there?

d) How many committees of three people have more Americans than Russians?

e) How many committees of three people do not have all three Americans?

27) If a coin is flipped five times, in how many different ways can it show up three heads?

28) To reach his destination, a man is to walk three blocks north and four blocks west. How many different routes are possible?

29) All three players of the women's beach volleyball team, and all three players of the men's beach volleyball team are to line up for a picture with all members of the women's team lined together and all members of men's team lined up together. How many ways can this be done?

30) From a group of 6 Americans, 5 Japanese and 4 German delegates, two Americans, two Japanese and a German are chosen to line up for a photograph. In how many different ways can this be done?

31) Find the fourth term of the expansion (2x - 3y)8.

32) Find the coefficient of the a5b4 term in the expansion of (a – 2b)9.