

1. [16 points] Solve this linear program graphically

Minimize  $Z = 50x + 40y$

Subject to C1:  $5x + 3y \geq 60$

C2:  $5x + 5y \geq 80$

C3:  $3x + 6y \geq 60$

C4:  $y \geq 2$

C5:  $x \geq 0$

x intercept

$(12, 0)$

$(16, 0)$

$(20, 0)$

y intercept

$(0, 20)$

$(0, 16)$

$(0, 10)$

horizontal line through  $(0, 2)$

(a) Graph and shade the feasible region.

(b) List ALL critical points.

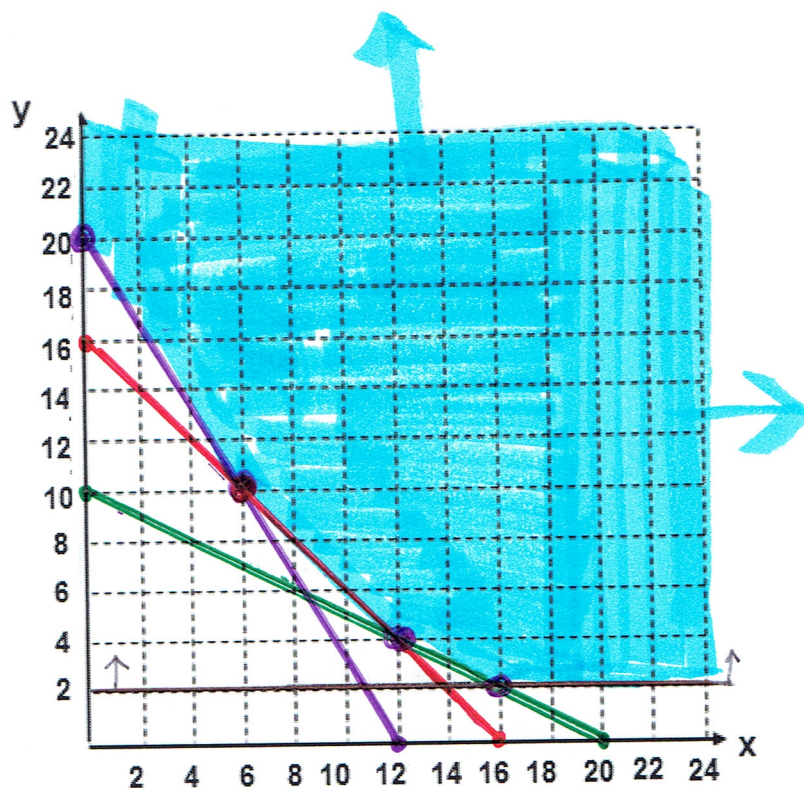
Critical Points

$(0, 20)$

$(6, 10)$

$(12, 4)$

$(16, 2)$



(c) Show all work to determine the optimal solution

Point

Z

$(20, 0)$

$Z = 50(0) + 40(20) = 800$

$(6, 10)$

$Z = 50(6) + 40(10) = 700 \leftarrow \text{MINIMUM}$

$(12, 4)$

$Z = 50(12) + 40(4) = 760$

$(16, 2)$

$Z = 50(16) + 40(2) = 880$

(d) State the values of the variables and the objective function for the optimal solution.

$x = 6$

$y = 10$

$Z = 700$

2. [24 points total] Solve the linear program using the simplex method.

$$\begin{aligned} \text{Maximize } Z &= 12x_1 + 18x_2 + 16x_3 \\ \text{subject to } C1: & x_1 + x_2 + x_3 \leq 160 \\ C2: & x_1 + x_2 + 3x_3 \leq 140 \\ C3: & x_1 + 2x_2 + 4x_3 \leq 240 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

- a. [6 points] Rewrite constraints as slack equations (equalities) with slack variable. State any additional constraints needed on the slack variables.

$$\begin{aligned} x_1 + x_2 + x_3 + y_1 &= 160 \\ x_1 + x_2 + 3x_3 + y_2 &= 140 \\ x_1 + 2x_2 + 4x_3 + y_3 &= 240 \end{aligned}$$

additional constraints are  $y_1 \geq 0$   $y_2 \geq 0$   $y_3 \geq 0$

- b. [8 points] Solve using the simplex method  
Write the initial simplex tableau. Show work to find the first pivot element.

$$\left[ \begin{array}{ccccccc|c} 1 & 1 & 1 & 0 & 0 & 0 & 160 \\ 1 & 1 & 3 & 0 & 1 & 0 & 140 \\ 1 & 2 & 4 & 0 & 0 & 1 & 240 \\ \hline -12 & -18 & -16 & 0 & 0 & 0 & 0 \end{array} \right] \begin{array}{l} 160/1=160 \\ 140/1=140 \\ 240/2=120 \end{array} \text{smallest ratio}$$

↑  
pivot column

First Pivot:  
Row 3  
Column 2

Use program APIVOT to perform the pivot operation.

Then write the simplex tableau after the first pivot and show the work to identify the next pivot element

$$\left[ \begin{array}{ccccccc|c} 1/2 & 0 & -1 & 1 & 0 & -1/2 & 0 & 40 \\ 1/2 & 0 & 1 & 0 & 1 & -1/2 & 0 & 20 \\ 1/2 & 1 & 2 & 0 & 0 & 1/2 & 0 & 120 \\ \hline -3 & 0 & 20 & 0 & 0 & 9 & 1 & 2160 \end{array} \right] \begin{array}{l} 40 \div \frac{1}{2} = 80 \\ 20 \div \frac{1}{2} = 40 \\ 120 \div \frac{1}{2} = 240 \end{array}$$

↑

Next Pivot:  
Row 2  
Column 1

- c. [10 points] Write Tableau #3 in the space below (after using APIVOT to perform the final pivot operation.)  
(If you used the procedure properly, it should be optimal.) Then in the answer box below, state the optimal value for each x variable, for each slack variable and for the objective function

$$\left[ \begin{array}{ccccccc|c} x_1 & x_2 & x_3 & y_1 & y_2 & y_3 & z \\ 0 & 0 & -2 & 1 & -1 & 0 & 20 \\ 1 & 0 & 2 & 0 & 2 & -1 & 40 \\ 0 & 1 & 1 & 0 & -1 & 1 & 100 \\ \hline 0 & 0 & 26 & 0 & 6 & 6 & 2280 \end{array} \right]$$

$x_1 =$  40  $x_2 =$  100  $x_3 =$  0  $y_1 =$  20  $y_2 =$  0  $y_3 =$  0  $Z =$  2280



3. [16 points total] **For this problem you need to set up the linear program in proper mathematical form and interpret the solution from the final matrix.**

Ace Home Security Systems installs 3 types of alarm systems: A, B, C

Each system requires 2 resources: "Design" and "Installation"

System A requires 2 hours to design and 3 hours to install.

System B requires 1 hour to design and 4 hours to install.

System C requires 1 hour for design and 2 hours to install.

They can sell at most a combined total of 50 of systems A and B each month, because they are more complicated than system C.

Each month there are at most 90 hours available for design and 250 hours available for installation.

$x_1$  = number of Alarm System A ;  $x_2$  = number of Alarm System B ;  $x_3$  = number of Alarm System C

System A sells for \$1200, System B sells for \$1000, and System C sells for \$500.

**How many of each type of system should they sell each month to maximize revenue from sales?**

**What is the maximum revenue?**

- a. [10 points] **WRITE THE LINEAR PROGRAM** using proper form and notation.

*State the objective function and what our optimization objective is.*

*Write all inequality constraints, including non-negativity constraints.*

**Do not do the simplex method to solve;** the results of the simplex method have been done for you already.

Maximize  $Z = 1200x_1 + 1000x_2 + 500x_3$  (Revenue)

Subject to  $2x_1 + x_2 + x_3 \leq 90$  (design)  
 $3x_1 + 4x_2 + 2x_3 \leq 250$  (installation)  
 $x_1 + x_2 \leq 50$  (demand for A, B)

$x_1 \geq 0$   $x_2 \geq 0$   $x_3 \geq 0$

- b. [6 points] The final tableau for the simplex method is given below.

**In complete sentences, clearly explain the optimal numbers of each type of alarm system and the optimal value of the objective function in context of the problem.**

*Just identifying the numbers for the answers without context will not get full credit.*

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	
0	0	1	$1/3$	$1/3$	$-5/3$	0	30
1	0	0	$-1/3$	$2/3$	$2/3$	0	10
0	1	0	$1/3$	$-2/3$	$1/3$	0	40
0	0	0	100	300	300	1	67000

They should sell 10 of system A, 40 of system B, and 30 of system C to obtain the maximum sales revenue of \$67,000.

4. [14 points total] For this problem you need to interpret the solution from the final matrix AND analyze the use of resources in the optimal solution.

The problem is already set up and solved for you.

PotzNPanz Co. makes three types of sets of cooking pots.

There are 4 resources needed to make the sets of pots.

The table shows how many units of each resource are needed for each type of set of pots.

It also shows the total number of units available of each resource.

The bottom row shows the profit per set for each type.

They want to know how much of each type of set of cooking pots to produce to maximize profit.

Resource	Set A	Set B	Set C	Resource Limits
Lid Making Resources	1 unit/set	2 unit/set	1 unit/set	29 units
Handle Making Resources	1 unit/set	1 unit/set	2 units/set	33 units
Pot Making Resources	4 units/set	3 units/set	2 units/set	47 units
Assembly Resources	2 units/set	2 units/set	1 unit/set	32 units
Profit per set	\$50	\$40	\$30	

$x_1$  = number of Pot Set A       $x_2$  = number of Pot Set B       $x_3$  = number of Pot Set C

The Linear Program is a "standard maximum" program.

Maximize  $Z = 50x_1 + 40x_2 + 30x_3$

Subject to

- $1x_1 + 2x_2 + 1x_3 \leq 29$  Constraint C1: Lid Resources
- $1x_1 + 1x_2 + 2x_3 \leq 33$  Constraint C2: Handle Resources
- $4x_1 + 3x_2 + 2x_3 \leq 47$  Constraint C3: Pot Resources
- $2x_1 + 2x_2 + 1x_3 \leq 32$  Constraint C4: Assembly Resources
- $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$

The simplex method has been done for you. Optimal Tableau after performing the Simplex Method is:

$x_1$	$x_2$	$x_3$	$y_1$	$y_2$	$y_3$	$y_4$	$Z$	
-7/4	0	0	1	1/4	-3/4	0	0	2
-1/4	0	1	0	3/4	-1/4	0	0	13
3/2	1	0	0	-1/2	1/2	0	0	7
-3/4	0	0	0	1/4	-3/4	1	0	5
5/2	0	0	0	5/2	25/2	0	1	670

C1: Lid Resources  
C2: Handle Resources  
C3: Pot Resources  
C4: Assembly Resources

- a. [6 points] Write the optimal solution in paragraph/sentence form in context of this problem, telling how many of each type of cooking pot sets should be produced and what the maximum profit is.

They should make no sets of Pot Set A, 7 sets of Pot Set B, and 13 sets of Pot Set C to maximize profit at \$670.

- b. [8 points] Analyzing use of resources:

Which resources (name them) are entirely used up in this production process?

Handle, Pot (because  $y_2=0, y_3=0$ )  
C2, C3 are satisfied as equality

Which resources are NOT entirely used up in this production process?

Name these resources and state how many units of each are not used ((how much is left over).

Lid  $y_1=2$  so 2 units lid resources are unused

Assembly  $y_4=5$  so 5 units of assembly resources are unused

Lid:  $1(0) + 2(7) + 1(13) = 27$  but limit is 29  
Assembly:  $2(0) + 2(7) + 1(13) = 27$  but limit is 32

TURN PAGE  
OVER FOR  
QUESTION  
#5



5. [10 points total]

**For this problem you only need to set up the linear program in correct mathematical form.**

*State the objective function and what our optimization objective is.  
Write all inequality constraints, including non-negativity constraints.  
Do NOT try to solve it.*

Giggle Big Media Co. is determining staffing needs for office Tech Support/Repair/Service. They hire 3 types of tech support/repair/service staff: Master, Technician, and Apprentice

**Write the linear program that could be used to determine how many of each type of staff they should schedule each day in order to minimize the total daily staffing cost.**

$x_1$  = number of Master ;  $x_2$  = number of Technicians ;  $x_3$  = number of Apprentices

They must repair/service at least 32 computers per day.

They are able to repair/service up to a limit of 40 computers per day.

Type of Staff	Master	Technician	Apprentice
Can service/repair ___ computers per day	4 computers	5 computers	3 computers

Note: A Master can handle only 4 computers/day because they get the more complicated repair jobs.

**Requirements for number of staff needed each day:**

There must be a combined total of at least 8 staff (all types)

There must be at least 2 Masters.

The number of Apprentices can be at most twice the number of Technicians.

Type of Staff	Master	Technician	Apprentice
Daily Pay Rate for staff	\$320 /day	\$240/day	\$160/day

$$\text{Minimize } Z = 320x_1 + 240x_2 + 160x_3 \quad (\text{cost})$$

$$\text{subject to } 4x_1 + 5x_2 + 3x_3 \geq 32$$

$$4x_1 + 5x_2 + 3x_3 \leq 50$$

$$x_1 + x_2 + x_3 \geq 8$$

$$x_1 \geq 2$$

$$x_3 \leq 2x_2$$

$$x_1 \geq 0 \quad x_2 \geq 0 \quad x_3 \geq 0$$