Math 11.07 Exam 1 Winter 2019 Name Last____________________ First____________________

1. [10 *points*] Solve the following system of linear equations using the **Gauss Jordan method.**

You must use the method requested to get credit. You must show work demonstrating the row operations needed to find the answer. If work does not use the correct method or does not connect to justify the answer, you will not get credit.

$$\begin{aligned} 3x + 9y + 6z &= 12 \\ y + 2z &= 4 \\ 2x + 2y + z &= 7 \end{aligned}$$

SHOW WORK! **Write instructions for row operations and the matrix resulting from each row operation to reach an identity matrix on the left to find the solution.** Write your answers in the answer box.

$$\left[\begin{array}{ccc|c} 3 & 9 & 6 & 12 \\ 0 & 1 & 2 & 4 \\ 2 & 2 & 1 & 7 \end{array}\right]$$

$R1 \div 3 \rightarrow R1$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 0 & 1 & 2 & 4 \\ 2 & 2 & 1 & 7 \end{array}\right]$$

$R3 - 2R1 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 3 & 2 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & -4 & -3 & -1 \end{array}\right]$$

$R1 - 3R2 \rightarrow R1$

$R3 + 4R2 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & -8 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 5 & 15 \end{array}\right]$$

$R3 \div 5 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -4 & -8 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R1 + 4R3 \rightarrow R1$

$R2 - 2R3 \rightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

or another possible way

$$\left[\begin{array}{ccc|c} 3 & 9 & 6 & 12 \\ 0 & 1 & 2 & 4 \\ 2 & 2 & 1 & 7 \end{array}\right]$$

$R1 - R3 \rightarrow R1$

$$\left[\begin{array}{ccc|c} 1 & 7 & 5 & 5 \\ 0 & 1 & 2 & 4 \\ 2 & 2 & 1 & 7 \end{array}\right]$$

$R3 - 2R1 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 7 & 5 & 5 \\ 0 & 1 & 2 & 4 \\ 0 & -12 & -9 & -3 \end{array}\right]$$

$R1 - 7R2 \rightarrow R1$

$R3 + 12R2 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -9 & -23 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 15 & 45 \end{array}\right]$$

$R3 \div 15 \rightarrow R3$

$$\left[\begin{array}{ccc|c} 1 & 0 & -9 & -23 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

$R1 + 9R3 \rightarrow R1$

$R2 - 2R3 \rightarrow R2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]$$

On the LEFT is the strategy we used in class

On the RIGHT is an alternate strategy Same number of steps but slightly harder numbers.

ANSWER: x = 4 y = -2 z = 3

Math 11.07 Exam 1 Winter 2019 Name Last ______________ First KEY

2. [12 *points total*] Tran is comparing sales jobs at two companies.
At company F, he can earn \$3000 per month plus 10% of his monthly product sales in dollars
At company G, he can earn \$2000 per month plus 15 % of his monthly product sales in dollars

a. [2 *points*] Write the linear functions y = f(x) and y = g(x) for companies F and G respectively that give y = monthly earnings in dollars, if x = monthly sales in dollars

Company F: $y = f(x) =$ $3000 + .10X$

Company G: $y = g(x) =$ $2000 + .15X$

b. [3 *points*] Show algebra work to find the monthly sales that gives the same (equal) earnings at both companies, and find the amount of earnings.

$$3000 + .10X = 2000 + .15X$$

$$1000 = .05X$$

$$X = \frac{1000}{.05} = 20{,}000$$

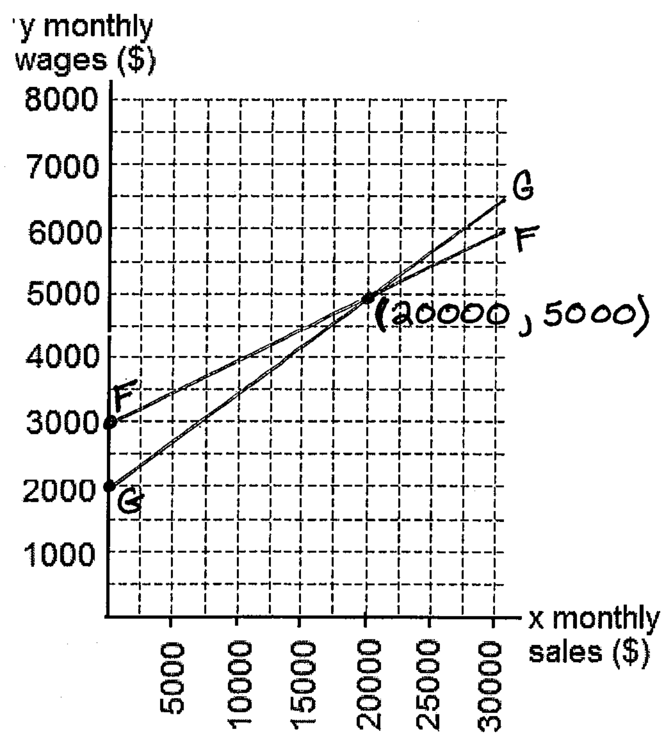
$$Y = 3000 + .10(20000) = 2000 + .15(20000) = 5000$$

ANSWER: The monthly earnings at both companies are equal if Tran sold \$ 20000 in sales and he would earn \$ 5000 per month.

c. [4 *points*]
Graph both functions on the grid provided. Draw them accurately.

Use a straight edge (ruler, edge of ID card) to get straight lines.

Label the (x,y) coordinates of the intersection point.



d. [3 *points*] Use your work in parts (b) and (c) to answer the following question:
For what interval of x values of monthly sales would Tran earn MORE at company G?

Company G would pay more if monthly sales are greater than \$20000.

Math 11.07 Exam 1 Winter 2019 Name Last________________ First________________

3. [9 *points total*] A company makes suitcases that sell for \$80 each.
The fixed production cost is \$66,000 and the variable cost is \$50 per suitcase.
Show work for all parts x = number of suitcases y is in dollars

a. [*4 points*] If 2000 suitcases are produced and sold, find the amount of profit or loss.

$R(x) = 80x \qquad R(2000) = 80(2000) = 160000$

$C(x) = 66000 + 50x \qquad C(2000) = 66000 + 100000 = 166000$

Profit = Revenue − Cost = 160,000 − 166000 = −6000

ANSWER: Circle one PROFIT or LOSS and fill in amount \$ 6000

b. [5 *points*] Find the break even point (amount and cost and revenue).

$66000 + 50x = 80x \qquad R(2200) = 80(2200) = \$176,000$

$66000 = 30x \qquad C(2200) = 66000 + 50(2000) = \$176000$

$\frac{66000}{30} = x$

x = 2200 suitcases

ANSWER: Break Even Quantity 2200 suitcases Cost \$ 176000 Revenue \$ 176000

4. [12 *points total*] A tee shirt company sells 400 tee shirts are sold if the price is \$20.
If the price is \$14, then 640 tee shirts are sold.
Show work for all parts. x = price (\$) y = quantity

a. [4 *points*] Find the equation for the demand function y = D(x) for quantity in terms of price.

x (\$)	y shirts
20	400
14	640

$m = \frac{640-400}{14-20} = \frac{240}{-60} = -40$

$y - 400 = -40(x-20)$

or $y - 640 = -40(x-14)$

simplifies to demand function

$y = D(x) = 1200 - 40x$

b. [3 *points*] The supply function is y = S(x) = 60x−250.
If the price is \$10, which is higher, supply or demand? By how much?

$S(10) = 60(10) - 250 = 350$

$D(10) = 1200 - 40(10) = 800$

ANSWER: Which is higher? Circle one: SUPPLY DEMAND BOTH EQUAL

If not equal, how much is the difference? 550 tee shirts

c. [5 *points*] Find the equilibrium quantity and price.

$60x - 250 = 1200 - 40x \qquad S(14.50) = 60(14.50) - 250$

$100x = 1450 \qquad D(14.50) = 1200 - 40(14.50)$

$x = \$14.50$ both equal 620 shirts

ANSWER: Equilibrium Quantity 620 tee shirts Equilibrium Price \$ \$14.50

round to dollars and cents

Math 11.07 Exam 1 Winter 2019 Name Last______________________First______________

5. [5 *points*] Don's Diner finds that they sell more soup when the weather is colder..
When the high temperature for the day is 60 °F they sell 25 quarts of soup.
If the daily high temperature decreases by 4 °F, they sell an additional 2 quarts of soup.

x = temperature in degrees Fahrenheit (°F) y = soup sales, in quarts

Find the equation that expresses y = soup sales as a function of (in terms of) x = temperature. *Show work.*

$(x_1, y_1) = (60°, 25 \text{ qts})$

$m = \frac{\Delta y}{\Delta x} = \frac{\Delta \text{ soup(qts)}}{\Delta \text{ temp }(°)} = \frac{+2}{-4} = -\frac{1}{2} = -.5$

$y - 25 = -.5(x - 60)$ simplifies to $y = 55 - .5x$

---

6. [5 *points*] A company produces fiber optic cable. x = amount of cable, in meters y is in dollars
It costs $8,000 to produce 1000 meters and it costs $12,500 to produce 4000 meters.

Find the fixed and variable costs and write the cost function y = C(x) that gives the total cost of producing this cable as a function of the amount produced. *Show work.*

x m	y $
1000	8000
4000	12500

$m = \frac{12500 - 8000}{4000 - 1000} = \frac{4500}{3000} = 1.50$

$y - 8000 = 1.5(x - 1000)$
simplifies to
$y = 6500 + 1.5x$

ANSWER: Variable Cost: 1.50x Fixed Cost: 6500 Cost Function y = C(x) = 6500 + 1.50x

7. [4 *points*] A store sells tee shirts and jeans at 2 locations: downtown in the city and in a shopping mall.
Sales of tee shirts and jeans in on Thursday and Friday at each location are shown in the matrices below.

Thursday Sales: $\mathbf{T} = \begin{array}{cc} \begin{array}{cc} \textit{mall} & \textit{city} \end{array} & \\ \begin{bmatrix} 20 & 30 \\ 25 & 15 \end{bmatrix} & \begin{array}{l} \textit{tee shirts} \\ \textit{jeans} \end{array} \end{array}$ Friday Sales: $\mathbf{F} = \begin{array}{cc} \begin{array}{cc} \textit{mall} & \textit{city} \end{array} & \\ \begin{bmatrix} 50 & 40 \\ 35 & 25 \end{bmatrix} & \begin{array}{l} \textit{tee shirts} \\ \textit{jeans} \end{array} \end{array}$

Also, on Saturday, they sold twice as much as they sold on Thursday.
Find matrix C= 2T − F which gives the change in sales between Friday and Saturday.

$$C = 2\begin{bmatrix} 20 & 30 \\ 25 & 15 \end{bmatrix} - \begin{bmatrix} 50 & 40 \\ 35 & 25 \end{bmatrix} = \begin{bmatrix} 2(20)-50 & 2(30)-40 \\ 2(25)-35 & 2(15)-25 \end{bmatrix} = \begin{bmatrix} -10 & 20 \\ 15 & 5 \end{bmatrix}$$

---

8. [5 *points*] **Find matrix product AB** (*your answer will have letters in it*).

$\mathbf{A} = \begin{bmatrix} -2 & 6 \\ a & c \\ 4 & 3 \end{bmatrix}$ $\mathbf{B} = \begin{bmatrix} 1 & 5 \\ 2 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} -2(1)+6(2) & -2(5)+6(3) \\ a(1)+c(2) & a(5)+c(3) \\ 4(1)+3(2) & 4(5)+3(3) \end{bmatrix}$$

$$AB = \begin{bmatrix} 10 & 8 \\ a+2c & 5a+3c \\ 10 & 29 \end{bmatrix}$$

Math 11.07 Exam 1 Winter 2019 Name Last________________________First________________

9. [*9 points total*] Each matrix below shows the result of using row operations with Gauss Jordan elimination to solve a system of three linear equations in three variables x,y,z.

	A	B	C	D
Linear System	$2x + 3y + 4z = -12$ $x - y + 4z = 29$ $5x + y + 3z = 61$	$x - 4y + 26z = -9$ $3x + y = 25$ $2x - y + 10z = 10$	$2x - y + 6z = 14$ $x - 2y - 15z = -12$ $x + 3y + 45z = 22$	$2x + 3y + 4z = 21$ $x - y + 4z = 13$ $5x + y + 3z = -2$
Augmented matrix after row operations	$A=\left[\begin{array}{ccc\|c} 1 & 0 & 0 & 15 \\ 0 & 1 & 0 & -14 \\ 0 & 0 & 1 & 0 \end{array}\right]$	$B=\left[\begin{array}{ccc\|c} 1 & 0 & 2 & 7 \\ 0 & 1 & -6 & 4 \\ 0 & 0 & 0 & 0 \end{array}\right]$	$C=\left[\begin{array}{ccc\|c} 1 & 0 & 9 & 0 \\ 0 & 1 & 12 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$	$D=\left[\begin{array}{ccc\|c} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 5 \end{array}\right]$

a. [3 *points*] Which system of equations has **NO solution**? C

b. [3 *points*] Which system of equations has **infinitely many solutions**? B

c. [3 points] For the system above that has infinitely many solutions, find any two specific solutions:

Solution 1: x = 7 y = 4 z = 0

Solution 2: x = 5 y = 10 z = 1

Workspace for part (c):

$x + 2z = 7$ so $x = 7 - 2z$ If $z = 0$, then $x = 7$, $y = 4$

$y - 6z = 4$ so $y = 4 + 6z$ If $z = 1$, then $x = 5$ $y = 10$

Many other possible answers: If $z = 2$, then $x = 7 - 2(2) = 3$ $y = 4 + 6(2) = 16$

Some of these are: If $z = -1$, then $x = 7 - 2(-1) = 9$ $y = 4 + 6(-1) = -2$

10. [10 *points*]
$$\begin{aligned} x + 3y + 3z &= 10 \\ x + y + 2z &= 2 \\ 2x + 2z &= 6 \end{aligned}$$

Write the system of linear equations as a MATRIX EQUATION AX=B
Then solve the system using the inverse matrix

Instructions: You must show the following work:

- write out the matrix equation.
- write the inverse matrix and show what calculations you need to do to find the solution.
- write the matrix giving the final solution resulting from of your work and write answer in the answer box.

IN THIS PROBLEM YOU CAN USE YOUR CALCULATOR TO PERFORM ANY MATRIX CALCULATIONS NEEDED.!
You must use the requested method. Do NOT solve using an augmented or row operations
No correct work using requested method = no credit, even if final answer is correct.

$$\begin{bmatrix} 1 & 3 & 3 \\ 1 & 1 & 2 \\ 2 & 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 2 \\ 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -3 & 1.5 \\ 1 & -2 & .5 \\ -1 & 3 & -1 \end{bmatrix}$$

$$\begin{aligned} AX &= B \\ A^{-1}AX &= A^{-1}B \\ IX &= A^{-1}B \\ X &= A^{-1}B \end{aligned}$$

$$X = A^{-1}B = \begin{bmatrix} 1 & 3 & 1.5 \\ 1 & -2 & .5 \\ -1 & 3 & -1 \end{bmatrix}\begin{bmatrix} 10 \\ 2 \\ 6 \end{bmatrix} = \begin{bmatrix} 13 \\ 9 \\ -10 \end{bmatrix}$$

ANSWER	x = 13	y = 9	z = -10